# Distributed Query Processing 

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#### Abstract

In this paper, various techniques for optimizing queries in distributed databases are presented. Although no attempt is made to cover all proposed algorithms on this topic, quite a few ideas extracted from existing algorithms are outlined. It is hoped that largescale experiments will be conducted to verify the usefulness of these ideas and that they will be integrated to construct a powerful algorithm for distributed query processing.

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## INTRODUCTION

Many algorithms to process queries in different distributed database systems have been proposed and implemented. In this paper, we restrict our attention to those strategies on relational databases [Codd 1970, 1972; Date 1977; Ullman 1980]. In spite of the restriction, there are numerous algorithms on the subject [Apers et al. 1983; Baldissera et al. 1979; Bernstein and Chiu 1981; Bernstein et al. 1981; Black and Luk 1982; Chang 1982a, 1982b; Cheung 1981; Chiu 1980; Chiu and Ho 1980; Epstein et al. 1978; Goodman et al. 1979; Gouda and Dayal 1981; Hevner 1980; Hevner and Yao 1979; Jarke and Koch 1983; Jarke and Schmit 1982; Kambayashi et al. 1982; Kerchberg et al. 1980; Kim 1982; Reiner 1982; Williams et al. 1981; Wong 1977, 1981; Yu and Ozsoyoglu 1979; Yu et al.

1982a, 1983, 1984a, 1984b], but they are not designed for the same environment. For example, the algorithm in Gouda and Dayal [1981] is suitable for a local network, the algorithm in Kerchberg et al. [1980] is designed for a star network, and most of the other algorithms are designed for long haul networks. Also, some environments have no fragmented relations, whereas in others some relations may be fragmented. In some situations, a query is embedded in a program and is likely to be executed repeatedly and therefore requires an extremely efficient strategy to process the query, even if the compilation cost is high. In other situations, the queries are submitted by users on an ad hoc basis, and thus a reasonably efficient strategy produced by a fast algorithm is needed. Because of this diversity, it is unlikely that a particular algorithm is suitable for all environments. In fact, no

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large-scale experiments have been performed to demonstrate the superiority of one algorithm over all other algorithms in a given environment.
Our intention is to present some of the important ideas that have been proposed for processing queries in distributed relational systems. The ideas involve the following: the necessity for and the assumptions used in estimating the sizes of temporary relations that are created during the processing of a distributed query; the use of semijoins to reduce intersite communication cost; the separation of an algorithm based on semijoins into three phases-the copy identification phase, the reduction phase, and the assembly phase; the characterization of queries solvable by semijoins; the transformation of cyclic queries into tree queries; the optimal processing of certain restricted types of queries, enhancements of semijoin strategies in
the reduction phase, and the identification of relations that need not participate in joins in the assembly phase; and the handling of fragments. These ideas are explored in the sections below.

No attempt is made to cover all proposed algorithms. Brief descriptions of some query-processing algorithms can be found in Reiner [1982]. Other issues in distributed databases can be found in Adiba et al. [1977], Rothnie and Goodman [1977a, 1977b], and Rothnie et al. [1980].

## 1. OPERATIONS AND COST MEASURES

In this paper a relational database [Codd 1970, 1972; Date 1977; Ullman 1980] with relations distributed in different sites is assumed. A relation is a two-dimensional table and is denoted by $R[X]$, where $X$ is the schema of relation $R$ and represents the names of columns. The relational data manipulation operations used in this paper are projection, selection, join, and semijoin [Bernstein and Chiu 1981]. They are described as follows:

Projection. The projection of relation $R$ on a set of attributes $T$ is denoted by $R . T$ or $R(T)$, where $R$ is a relation with schema $X$, and $T$ is a subset of $X$. It is obtained by discarding all columns of $R$ that are not in $T$, and eliminating duplicated rows, if necessary.

Selection. The selection of those tuples whose $A$-attribute values equal to a specified constant in relation $R$ is denoted by ( $R . A=$ the specified constant), where $A$ is an element of $X$.

It is obtained by choosing all rows of $R$ whose $A$-attribute values are equal to the specified constant. One or more select clauses on the same relation may be used in selection. Operators other than " $=$ " (e.g., $\geq$ and $\neq$ ) are allowed.

Join. The join of relation $R_{1}$ with relation $R_{2}$ on attribute $A$ is denoted by ( $R_{1} . A$ $=R_{2} . A$ ), where $R_{1}$ and $R_{2}$ are the joining relations. Let $X$ and $Y$ be the schema of $R_{1}$ and $R_{2}$, respectively. The attribute $A$, which is an element of $X$ and $Y$, is the joining attribute of $R_{1}$ and $R_{2}$.

The join is obtained by concatenating each row of $R_{1}$ with each row of $R_{2}$ whenever the $A$-attribute values of the two rows

(a)

(b)

Figure 1. To answer a query, (a) $X$ units of data are transferred from site 1 to site 2 and $Y$ units of data are transferred from site 2 to site 3. (b) $X$ units of data from site 1 and $Z$ units of data from site 2 are transferred in parallel to site 3.
are equal. Since the equality operation results in two identical columns, one column may be eliminated. One commonly used join operation is the natural join, where two rows from the joining relations are concatenated whenever the corresponding values under all common attributes of the two relations are equal. We use ( $R_{1}=R_{2}$ ) to denote the natural join of relations $R_{1}$ and $R_{2}$.

Semijoin [Bernstein and Chiu 1981; Bernstein and Goodman 1979; Yu and Ozsoyoglu 1979]. The semijoin from relation $R_{2}$ to relation $R_{1}$ on attribute $A$ is denoted by $R_{2}-A \rightarrow R_{1}$, where $R_{2}$ is the sending relation, $R_{1}$ is the reduced relation, and $A$ is the joining attribute. Sometimes we use $R_{2} \rightarrow R_{1}$ to represent $R_{2}-A \rightarrow R_{1}$ if there is no need to identify the attribute. It can be obtained by joining $R_{1}$ and $R_{2}$ on attribute $A$, then projecting the resulting relation on the schema of $R_{1}$. Semijoins are also useful in database machines (see, e.g., Babb [1979]).

If no relation is fragmented, then in the performance of projections and selections, a local processing cost only is involved. However, when joins and semijoins are executed, communication costs between different sites may be incurred in addition to the local processing cost. For example, if $R_{1}$ and $R_{2}$ are in different sites, $R_{1}$ must be sent to the site containing $R_{2}$, or $R_{2}$ must be sent to the site of $R_{1}$ before the operation can take place.

Local processing costs usually are evaluated in terms of the number of disk accesses
and CPU processing time, while communication costs are expressed in terms of the total amount of data transmitted. For geographically dispersed computer networks, communication cost is normally the dominant consideration, but local processing cost is of greater significance for local networks. In this paper, we are mostly concerned with geographically dispersed computer networks.

We assume that the cost of transferring an amount of data, say $X$, from one site to another site is $c_{0}+c_{1} * X$, where $c_{0}$ is the start-up cost of initiating transmission and $c_{1}$ is a proportionality constant. The cost for answering a query can be expressed in terms of the total cost measure or the response time measure. The total cost measure [Hevner and Yao 1979] is the sum of the costs of transferring data. In Figure 1a, where $X$ units of data necessary to answer a query is transferred from site 1 to site 2 and $Y$ units of data from site 2 to site 3 , the total cost is $\left(c_{0}+c_{1} * X\right)+\left(c_{0}+c_{1} * Y\right)$ $=2 c_{0}+c_{1}(X+Y)$. The response time measure [Hevner and Yao 1979] is the time from the initiation of the query to the time when the answer is produced. In Figure 1b, where $X$ units of data from site 1 and $Z$ units of data from site 2 are transferred in parallel to site 3 to answer the query, the response time cost is the maximum of $c_{0}+$ $c_{1} * X$ and $c_{0}+c_{1} * Z$. In this paper, we are mostly concerned with the total cost measure only.

Since the amount of data transferred affects the cost of a strategy, attempts have

| R1 |  | R2 |  |
| :---: | :---: | :---: | :---: |
| A | B | B | c |
| 01 | b1 | b1 | c1 |
| 02 | b1 | b2 | c2 |
| 02 | b3 | b5 | c1 |
| 02 | b4 | b5 | c2 |
| 03 |  | b6 | c4 |
|  |  | b7 | c2 |
|  |  | b8 | c3 |

$R 2-B \rightarrow R 1$

(a)

R1 R2

$\mathbf{R 1}-\mathbf{B} \rightarrow \mathbf{R 2}$

(b)

Figure 2. Illustrating semijoms.
been proposed to reduce it. A promising approach is to make use of semijoins [Bernstein and Chiu 1981; Hevner and Yao 1979]. For example, the semijoin from relation $R_{2}[B, C]$ to relation $R_{1}[A, B]$ on attribute $B\left(R_{2}-B \rightarrow R_{1}\right)$ can be obtained by projecting $R_{2}$ on attribute $B$, then joining the result of the projection with $R_{1}$, rather than computing the join and then the projection. We can easily see that $R_{2}-B \rightarrow$ $R_{1}$ is never bigger than $R_{1}$, and is usually much smaller in size. In Figure 2a, the attribute values $\left\{b_{3}, b_{4}\right\}$ of $R_{1}$ do not appear in $R_{2}$. Thus the corresponding tuples $\left\{\left(a_{2}, b_{3}\right),\left(a_{2}, b_{4}\right),\left(a_{3}, b_{3}\right)\right\}$ are eliminated from $R_{1}$. The semijoin from $R_{2}$ to $R_{1}$ consists of the retained tuples $\left\{\left(a_{1}, b_{1}\right)\right.$, $\left.\left(a_{2}, b_{1}\right)\right\}$.

Suppose that $R_{1}$ and $R_{2}$ as given in Figure 2 are at different computer sites, and the join of $R_{1}$ and $R_{2}$ is desired at the site containing $R_{1}$. Suppose that each value in
each of the attributes $A, B$, and $C$ has unit width. To obtain a join of $R_{1}$ and $R_{2}$ at the site containing $R_{1}$, one method is that we send $R_{2}$ to $R_{1}$, then take the join at the site containing $R_{1}$. This method has a communication cost of $c_{0}+c_{1}[7(1+1)]=c_{0}+$ $14 c_{1}$. The second method consists of sending the $B$-attribute values of $R_{1}$, that is, $\left\{b_{1}, b_{3}, b_{4}\right\}$ to the site containing $R_{2}$. Then all those tuples of $R_{2}$ whose $B$-attribute values do not appear in $\left\{b_{1}, b_{3}, b_{4}\right\}$ are eliminated; that is, $R_{1}-B \rightarrow R_{2}$ is computed. This operation yields $\left\{\left(b_{1}, c_{1}\right)\right\}$. The reduced $R_{2}$ is then sent back to the site containing $R_{1}$ to join with $R_{1}$. In this example, the sending of $R_{1}$ projected on $B$ costs $c_{0}+c_{1}(3)$. The sending of the reduced $R_{2}$ costs $c_{0}+c_{1}(1+1)$. Thus the second method is better than the first if $2 c_{0}+5 c_{1}$ $<c_{0}+14 c_{1}$. This use of semijoin is justified in situations of small $c_{0}$. On the other hand, if the number of $B$-attribute values in com-
mon between $R_{1}$ and $R_{2}$ is large, as in Figure 2 b , the use of semijoin may not be profitable. Clearly, it is desirable to estimate the number of attribute values in common between two relations before deciding to execute certain semijoins.

## 2. ESTIMATION

It can be inferred from the previous section that the performance of a distributed query-processing algorithm depends to a significant extent on the estimation algorithm used to evaluate the expected sizes of some intermediate relations. The choice of a reasonable estimation algorithm is therefore extremely important, as is described below.

Suppose that relations $R_{1}$ and $R_{2}$ are single-attribute relations, and, further, that the values of the common attribute, say $A$, are uniformly and independently distributed on the relations. The desired estimation is the size of $R_{2}-A \rightarrow R_{1}$, that is, $\left|R_{2}-A \rightarrow R_{1}\right| * w$, where $|X|$ denotes the cardinality of $X$ and $w$ is the average width of a tuple in $R_{1}$.

Letting $p_{l a}$ be the probability that a value in attribute $A$ appears in $R_{i}, i=1,2$, then $p_{\text {ra }}$ is called the selectivity of $R_{ı}$ on attribute $A$. Since the values in the two relations are independently distributed, the probability that a value appears in both relations is $p_{1 a} * p_{2 a}$. Thus the expected number of distinct values in common between the two relations is $|A| * p_{1 a} * p_{2 a}$, where $|A|$ is the cardinality of the domain of the attribute $A$. The size of the reduced $R_{1}$ can be estimated to be $|A| * p_{1 a} * p_{2 a} * w$. This can be rewritten as $\left|R_{1}\right| * p_{2 a} * w$.

In a different scenario, $R_{2}$ is the same as above but $R_{1}$ is a relation with two attributes $A$ and $B$. After the semijoin, $R_{2}-A$ $\rightarrow R_{1}$, the cardinality of $R_{1}$ can be estimated as $\left|R_{1}\right| * p_{2 a}$, where $\left|R_{1}\right|$ is the number of tuples of $R_{1}$ before the semijoin was performed. The estimation problem of the cardinality of $R_{1}$ projected on the $B$-attribute after the semijoin can be demonstrated in the following ball-color problem: "There are $n$ balls with $m$ different colors. Find the expected number of colors if $t$ balls are randomly selected from the $n$ balls." The
correspondences are as follows: $n$ balls are the number of tuples of $R_{1}$ before the semijoin, $m$ colors are the number of distinct values of $R_{1}$ projected on the $B$-attribute before the semijoin, and the $t$ selected balls correspond to the number of tuples of $R_{1}$ after the semijoin. The expected number of colors of the $t$ selected balls is
$g(m, n, t)$
$=m *\left[1-\prod_{i=1}^{t}\left(\frac{n((m-1) / m)-i+1}{n-i+1}\right)\right]$.
This solution is the same as the solution given by Yao [1977] in the block-access problem. It should be pointed out that, although $t$ is a parameter given in the ballcolor problem, the number of tuples of $R_{1}$ after the semijoin needs to be estimated. Some inaccuracy in the estimation can be expected. The formula, if evaluated in the present form, is computationally expensive and may cause overflow or underflow for large values of $t$. The following function given in Goodman et al. [1979] and Bernstein et al. [1981] is an approximation to the formula described above:

$$
\begin{array}{ll}
m & \text { if } \quad t \geq 2 m \\
\frac{(t+m)}{3} & \text { if } \quad 2 m \geq t \geq\left(\frac{m}{2}\right) \\
t & \text { if } \quad\left(\frac{m}{2}\right)>\mathrm{t}
\end{array}
$$

A semijoin strategy can be viewed as a directed graph, where the vertices are the relations and a directed edge from $R_{i}$ to $R_{j}$; that is, $R_{\mathrm{t}} \rightarrow R_{\text {g }}$ denotes the semijoin from $R_{t}$ to $R_{\text {, }}$. The semijoins that are executed first are those involving nodes with indegree $=0$. For example, in the semijoin strategy $R_{t} \rightarrow R_{j} \rightarrow R_{k}, R_{\imath}$ has in-degree $=$ 0 and the semijoin $R_{i} \rightarrow R_{j}$ is executed first. After the execution of the semijoin, the reduced $R_{j}$, denoted by $R_{J^{\prime}}$, is produced. The strategy becomes $R_{j}, \rightarrow R_{k} . R_{j^{\prime}}$ has in-degree $=0$, implying that the semijoin $R_{J^{\prime}} \rightarrow R_{k}$ will be executed next. Clearly, directed cycles will not appear in a valid semijoin strategy; otherwise, the semijoin strategy does not terminate.

Now, suppose that $R_{1}$ and $R_{2}$ are the same as above, and $R_{3}$ is a single-attribute relation with attribute $B$. After $R_{1}$ is reduced by $R_{2}$ and $R_{3}$ using the semijoins $R_{2}$ $A \rightarrow R_{1} \leftarrow B-R_{3}$, the number of tuples of the resulting $R_{1}$ can be estimated as $p_{2 a} * p_{3 b} *\left|R_{1}\right|$, where $p_{3 b}$ is the selectivity of $R_{3}$ under attribute $B$. Thus the size of $R_{1}$ can be estimated as $p_{2 a} * p_{3 b} *\left|R_{1}\right| * w$, where $w$ is the average width of a tuple in $R_{1}$. Moreover, the expected number of distinct $A$ values and the expected number of distinct $B$ values in $R_{1}$ can be estimated by using the block-access formula as described above.

We are given three relations $R_{1}, R_{2}$, and $R_{3}$, each having the attribute values $A$ and $B$. In the following strategies, $R_{1}-B \rightarrow$ $R_{2}-A \rightarrow R_{3}$ and $R_{2}-A \rightarrow R_{1}-B \rightarrow$ $R_{3}, R_{3}$ is reduced by the same set of relations on the same attributes. In the first case, the number of distinct $A$-values of $R_{2}$ after executing the semijoin $R_{1}-B \rightarrow R_{2}$ is estimated to be $g\left(p_{2 a}|A|,\left|R_{2}\right|\right.$, $\left.\left|R_{2}\right| p_{1 b}\right)$. Thus the number of tuples of $R_{3}$ can be estimated to be $\left|R_{3}\right| * g\left(p_{2 a}|A|\right.$, $\left.\left|R_{2}\right|,\left|R_{2}\right| p_{1 b}\right) /|A|$. In the latter case, the number of tuples of $R_{3}$ can be estimated to be $\left|R_{3}\right| * g\left(|B| p_{1 b},\left|R_{1}\right|,\left|R_{1}\right| p_{2 a}\right) /|B|$. Since the two expressions are in general not equal, the reduced relations $R_{3}$ are different in size for the two strategies after the execution of the semijoins. Thus estimating the size of a relation in a semijoin strategy necessitates recognizing the history of the operations. Such estimation algorithms are given by Bernstein et al. [1981], Luk and Black [1981], and Yu et al. [1983]. The above estimation techniques may be extended to apply to multiattribute semijoins.

Consider the semijoin $R_{1}-A B \rightarrow R_{2}$, where both $R_{1}$ and $R_{2}$ contain attributes $A$ and $B$ and $A B$ denotes the composite attribute $A$ and $B$. Letting $p_{a b}$ be the selectivity of $R_{i}$ under $A B, i=1$ or 2 , define it as $\left|R_{\imath}(A, B)\right| /(|A| *|B|)$, where $\left|R_{\imath}(A, B)\right|$ is the number of tuples in the projection of $R_{t}$ on $A B$. Then the number of tuples in the resulting $R_{2}$ can be evaluated as $p_{1 a b}\left|R_{2}\right|$, and the expected number of distinct values of $R_{2}$ under $A$ can be estimated via the ballcolor problem with $n=\left|R_{2}(A, B)\right|, t=$ the
size of $R_{2}(A, B)$ after the semijoin $=$ $p_{t a b}\left|R_{2}(A, B)\right|$, and $m=\left|R_{2}(A)\right|$.

We have mentioned that if $R_{t}$ and $R_{J}$ are at different sites, $R_{i}-A \rightarrow R_{j}$ can be computed by sending $R_{i}(A)$ from the site containing $R_{i}$ to the site containing $R_{j}$. Other methods for computing the semijoin have been developed [Kambayashi 1981, 1982; Krishnamurthy and Morgan 1984; Sacco 1984; Wah and Lien 1984; Yu et al. 1982b]. For example, if $\left|R_{l}(A)\right|<|A|-$ $\left|R_{\imath}(A)\right|$, it is cheaper to send the complement, $A-R_{i}(A)$. Another way is to send a bit vector indicating the presence or absence of the attribute values. These data compression techniques help to reduce data transfer.

## 3. THREE PHASES

We shall concern ourselves with strategies of semijoins in Sections 3-7 of this paper. The queries under consideration are of the form \{target component|qualification component\}, where the qualification component identifies the tuples of the relations satisfying the query and is of the form AND ( $R_{r} \cdot A_{k}=R_{j} . A_{l}$ ); that is, it is the conjunction of equality clauses where the $R$ 's stand for the relations and the $A$ 's for the attributes. The target component specifies the attributes of certain relations to be outputted to the user and is of the form ( $R_{t} . A_{s}, \ldots$, $R_{g} . A_{n}$ ) as in the following example: $\left\{\left(R_{1} . A_{1}, R_{3} . A_{3}\right) \mid\left(R_{1} . A_{1}=R_{2} . A_{1}\right)\right.$ AND ( $R_{1} \cdot A_{2}=R_{3} . A_{2}$ ) \}. For each tuple of $R_{1}$, each tuple of $R_{2}$, and each tuple of $R_{3}$ satisfying ( $R_{1} \cdot A_{1}=R_{2} \cdot A_{1}$ ) AND ( $R_{1} \cdot A_{2}=R_{3} \cdot A_{2}$ ), the tuple of $R_{1}$ and the tuple of $R_{3}$ are projected onto attributes $A_{1}$ and $A_{3}$, respectively, to be presented to the user.

If a clause of the form ( $R_{2} . A_{k}=$ constant) appears in the qualification of the query, this clause can be processed at the local site containing $R_{i}$ and therefore can be eliminated. If the qualification of a query is a disjunction of equality clauses, then each clause can be treated as the qualification of a subquery. After evaluating the subqueries, the results are merged to provide the answer to the original query. We shall restrict ourselves to the type of queries mentioned above.

Query: $\{(R 1 . A 1, R 3 . A 3) \mid(R 1 . A 1=R 2 . A 1) A N D(R 1 . A 2=R 3 . A 2)\}$


Figure 3. Three relations distributed on three sites.

The processing of distributed queries can be separated into three phases: the copy identification phase, the reduction phase, and the assembly phase. In the copy identification phase, one or more copies of every relation appearing in the qualification of the query are identified and will be used to process the query. Since a distributed database system may contain duplicate copies of some relations, the identification of appropriate copies of the relations in order to minimize the cost of transmission may not be an easy process.

The following query will serve as an example: $\left\{\left(R_{1} \cdot A_{1}, R_{3} \cdot A_{3}\right) \mid\left(R_{1} \cdot A_{1}=R_{2} \cdot A_{1}\right)\right.$ AND ( $R_{1} . A_{2}=R_{3} . A_{2}$ ) ), with the three relations $R_{1}, R_{2}$, and $R_{3}$ distributed in the three sites $S_{1}, S_{2}$, and $S_{3}$, as shown in Figure 3. One way of approaching the problem would be to select the copy of $R_{1}$ from $S_{2}$, the copy of $R_{2}$ from $S_{1}$, and the copy of $R_{3}$ from $S_{3}$, but this operation would likely incur high transmission cost, because two of the three relations have to be sent to the site containing the other relation. Another method would be to have the copies of the relations $R_{2}$ and $R_{3}$ in site $S_{3}$ and the copy of the relation $R_{1}$ in $S_{2}$. In this case, since $R_{2}$ and $R_{3}$ do not have a common joining attribute, either $R_{1}$ is sent to site $S_{3}$ or $R_{2}$ and $R_{3}$ are sent to $S_{2}$. Still another approach would be to have copies of $R_{1}$ and $R_{2}$ at site $S_{1}$ and the copy of $R_{3}$ at site $S_{3}$. This last choice is the best of the three, because (1) $R_{1}$ and $R_{2}$ projected on $A_{1}$ can be merged together without communication cost to produce a relation that is not larger than the original relation $R_{1}$, and (2) it is
then sufficient either to transfer the merged relation from $S_{1}$ to $S_{3}$ or to transfer $R_{3}$ from $S_{3}$ to $S_{1}$.
Suppose that a query references $n$ relations. If relation $R_{t}$ has $X_{i}$ copies, $1 \leq i \leq$ $n$, then a straightforward enumerative algorithm to choose one copy for each relation takes time $O\left(\prod_{\imath=1} x_{2}\right)$. This is exponential in time. It turns out [ Yu et al. 1982b] that finding one copy among several possibilities of each relation referenced by a given query so that the cost of answering the query is minimized is a NP-hard problem (in the number of sites having at least one copy of a relation referenced by the query). This situation holds true even when restricted to the simple queries (all relations have one and exactly the same attribute) in a fully connected network (i.e., each site can communicate directly with every other site).

In the reduction phase, semijoins are usually used to eliminate tuples of the relations that do not satisfy the qualification of the query. For example, for the query cited earlier with the best choice of the copies of the relations, one could perform semijoin $R_{2}-A_{1} \rightarrow R_{1}$ to eliminate some tuples of $R_{1}$ without incurring communication cost. If the result of the semijoin were to be $R_{1^{\prime}}$, other semijoins could then be performed $R_{1^{\prime}}-A_{2} \rightarrow R_{3}$ to reduce $R_{3}$, or semijoin $R_{3}-A_{2} \rightarrow R_{1^{\prime}}$ to further reduce $R_{1^{\prime}}$.

In the assembly phase, relations in the qualification component of the query are sent to one site to produce the output required by the user. For example, in the above query, $R_{1^{\prime}}$ (which is the result of

Query $=\{($ R1.A1,R2.A2 $) \mid($ R1.AI $=R 2 . A 4)$
AND (R1.A2 $=$ R3.A5) AND(R1.A1 $=$ R4.A5) $\}$

(a)


(b)

Figure 4. Representing a query by its jom-graph: (a) join-graph; (b) joingraph with attributes renamed.
semijoin $R_{2}-A_{1} \rightarrow R_{1}$ ) can be sent to $S_{3}$ to be merged with $R_{3}$ to produce the output.

We should like to point out that the separation of the query-processing strategy into the three phases may not yield the least transmission cost; rather, it tends to simplify the concepts involved.

A reasonable strategy for choosing the copies of the relations is to find the minimum number of sites containing chosen copies of the relations. Unfortunately, that is also a NP-complete problem [Yu et al. 1982b]. However, since the number of relations referenced by a query and the number of sites containing those relations is usually small, finding the number of sites by enumeration does not require much time. The reduction phase and the assembly phase will be described in more detail in the subsequent sections.

## 4. TREE QUERIES VERSUS CYCLIC QUERIES

### 4.1 Characteristics

Only certain types of queries can be solved using semijoins. More precisely, a relation appearing in the qualification of a query is said to be fully reduced if all tuples not satisfying the qualification of the query
have been eliminated. It is clear that if the joins of all the relations in the qualification are taken, and the resulting relation is then projected back onto the attributes of the original relations, then the projected relations will then be fully reduced, because any tuple of each projected relation not satisfying the qualification would have been eliminated by the joins. If semijoins are used to reduce relations, less communication cost may be incurred. However, depending on the type of query, the relations appearing in the query may not be fully reduced. As a result, communication cost in assemblying the relations can still be high. A precise characterization of the type of queries whose referenced relations can be fully reduced by semijoins is therefore desirable. The characterization is facilitated by defining a join-graph and a query-graph [Bernstein and Chiu 1981].

The vertices of a join-graph are described as $\left\{R_{i} . A_{j} \mid R_{i}\right.$ is a relation, $A$, is an attribute, and $R_{i} . A_{j}$, appears in a clause of the qualification\}. The edges of the graph represent the equality clauses. As shown in Figure 4a, each $R_{i} . A_{j}$ is a vertex, and an equality clause of the form ( $R_{i} . A_{J}=R_{k} . A_{1}$ ) is represented by an edge between $R_{t} . A_{\text {, }}$ and $R_{k} \cdot A_{1}$. Since equality is a transitive operator, $\left(R_{t} \cdot A_{j}=R_{k} \cdot A_{1}\right) \mathrm{AND}\left(R_{k} \cdot A_{1}=R_{t} . A_{m}\right)$

## Join-graph

R1.A2 ——R3.A2

Query - graph


Figure 5. An example of join-graph and query-graph.
imply ( $R_{r} . A_{j}=R_{t} \cdot A_{m}$ ). If two or more attributes of a relation are transitively related, it is sufficient to retain one of them, since the other can be eliminated by local processing.

We can thus rename all attributes that are transitively related to be the same attribute. In Figure $4 \mathrm{~b}, A_{1}, A_{4}$, and $A_{5}$ in one component of the join-graph are all renamed to be $A_{1}$, while $A_{2}$ and $A_{5}$ in another component are renamed to be $A_{2}$. In other words, all vertices in a connected component refer to the same joining attribute, and the connected component can be uniquely identified by the attribute. If $R_{\imath} . A_{k}$ and $R_{j} . A_{k}$ are in the same connected component identified by attribute $A_{k}$, then clearly $R_{t}-A_{k} \rightarrow R_{J}$ and $R_{J}-A_{k} \rightarrow R_{\imath}$ are possible semijoins. On the other hand, $R_{l} . A_{k} \rightarrow R_{f} . A_{t}$, for $t \neq k$, is not a possible semijoin because ( $R_{t} . A_{k}=R_{t} . A_{t}$ ) is neither stated nor implied by the qualification. In this manner all possible semijoins of the qualification are of the form $R_{\imath}-A_{k} \rightarrow R_{j}$ or $R_{s}-A_{k} \rightarrow R_{t}$ for some attribute $A_{k}$, after the renaming of the attributes.

The vertices of the query-graph are the relations appearing in the qualification. An edge ( $R_{\imath}, R_{\jmath}$ ) with label $A_{k}$ exists in the query-graph if ( $R_{r}, A_{k}=R_{\text {, }}, A_{k}$ ) is a clause in the qualification. If ( $R_{l} . A_{l}=R_{\text {, }} . A_{l}$ ) also appears in the join-graph, the label of the edge in the query-graph is $\left\{A_{k}, A_{l}\right\}$; that is, the label is to include all attribute names that participate in the clauses involving the relations $R_{i}$ and $R_{j}$. For example, Figure 5 illustrates a join-graph and its corresponding query-graph.
If a query-graph is a tree in the graphtheoretical sense, then it can be shown [Bernstein and Chiu 1981] that a sequence of semijoins can fully reduce all the relations. The sequence suggested by Bernstein
and Chiu [1981] is rather simple. A relation is chosen arbitrarily in the query-graph as the root of the tree, for example, $R$. Then the leaves of the tree are well defined. For example, in Figure 6 the leaves are $R_{2}, R_{5}$, $R_{6}$, and $R_{4}$. The process of fully reducing all relations consists of two phases: (1) "leaves to root," which fully reduces the root relation, and (2) "root to leaves," which fully reduces the other relations after carrying out Phase 1.

The "leaves to root" phase consists of taking semijoins from each relation to its parent, starting from the leaves and ending at the root. Semijoins from relations to their common parent all should be taken before any operation of the parent with its immediate ancestor is taken. For example, in Figure 6, the semijoins $R_{5}-A_{4} \rightarrow R_{3}$ and $R_{6}-A_{5} \rightarrow R_{3}$ are taken before the semijoin $R_{3}-A_{2} A_{3} \rightarrow R_{1}$ is executed. The clause ( $R_{5} . A_{4}=R_{3} . A_{4}$ ) is satisfied intuitively by $R_{3}$ after the semijoin $R_{5}-A_{4} \rightarrow$ $R_{3}$ is taken. Similarly, the clause ( $R_{3} \cdot A_{5}=$ $R_{6} . A_{5}$ ) is satisfied by $R_{3}$ after the execution of the semijoin $R_{6}-A_{5} \rightarrow R_{3}$. Thus $R_{3}$ satisfies the two clauses after application of the two semijoins. Similar arguments show that at the end of the first phase, the root relation $R_{1}$ will satisfy the clauses $\left\{\left(R_{5} \cdot A_{4}=R_{3} \cdot A_{4}\right),\left(R_{3} \cdot A_{5}=R_{6} \cdot A_{5}\right), \ldots\right.$, ( $R_{4} \cdot A_{4}=R_{1} \cdot A_{4}$ ); ; that is, the relation $R_{1}$ is fully reduced.
In the second phase ("root to leaves"), the fully reduced root relation, let us say $R$, is used to reduce its immediate descendants. When the semijoin from $R$ to an immediate descendant, for example, $R_{t}$, is taken, $R_{t}$ is fully reduced. This can be demonstrated by constructing a tree with $R_{\mathrm{t}}$ as the root. The "leaves to root" phase with $R_{i}$ as root will be executed after the completion of the "leaves to root" phase with $R$ as

## Query-groph



Join-graph



(a)

To fully reduce R1

$$
\begin{gathered}
(R 5-A 4-R 3, R 6-A 5 \rightarrow R 3, \\
R 2-A 1-R 1, R 3-A 2 A 3-R 1, \\
R 4-A 4-R 1)
\end{gathered}
$$



Query-graph

To fully reduce the other relations after R1 is fully reduced

$$
\begin{gathered}
(R 1-A 1-R 2, R 1-A 2 A 3-R 3, \\
R 1-A 4-R 4, R 3-A 4-R 5, \\
R 3-A 5-R 6)
\end{gathered}
$$

Figure 6. A sequence of semijoins fully reduces the relations.
root plus the semijoin $R \rightarrow R_{i}$. In Figure 6 , the sequence of semijoins $\left(R_{5}-A_{4} \rightarrow\right.$ $R_{3}, R_{6}-A_{5} \rightarrow R_{3}, R_{2}-A_{1} \rightarrow R_{1}, R_{3}-$ $A_{2} A_{3} \rightarrow R_{1}, R_{4}-A_{4} \rightarrow R_{1}$ ) (the sequence is the "leaves to root" phase with $R_{1}$ as root) followed by $R_{1}-A_{4} \rightarrow R_{4}$ contains the sequence of semijoins ( $R_{5}-A_{4} \rightarrow R_{3}$, $R_{6}-A_{5} \rightarrow R_{3}, R_{2}-A_{1} \rightarrow R_{1}, R_{3}-A_{2} A_{3}$ $\rightarrow R_{1}, R_{1}-A_{4} \rightarrow R_{4}$ ) (the sequence is the "leaves to root" phase with $R_{4}$ as root). Thus $R_{4}$ will be fully reduced. The process of using the newly fully reduced relation to fully reduce its immediate descendants is continued until all the leaves are reached; at this point, all relations are fully reduced. This process is illustrated in Figure 6.

This discussion should make clear that if the query-graph of the qualification of a query is a tree, the relations can be fully
reduced by semijoins. Even if the querygraph of a given qualification should be cyclic, an equivalent qualification exists that uses a tree query-graph, as is demonstrated in Figure 7a. The qualification is equivalent to that given in Figure 7b because ( $R_{1} \cdot A_{2}=R_{3} \cdot A_{2}$ ) AND ( $R_{3} \cdot A_{2}=$ $R_{4} \cdot A_{2}$ ) is equivalent to ( $R_{1} \cdot A_{2}=R_{3} \cdot A_{2}$ ) AND ( $R_{1}, A_{2}=R_{4} \cdot A_{2}$ ). The latter qualification has a tree query-graph and therefore is solvable by semijoins.

The definition of a tree query is that the query graph of its qualification or an equivalent qualification is a tree. A query is a cyclic query if none of the query-graphs of equivalent qualifications is a tree. As illustrated earlier, if the query is a tree query, the relations of a tree query can be fully reduced by semijoins, but semijoins may be
Join-graph
Query-graph
Equivalent Join-graph
Equivalent Query-graph

(a)

(b)

Figure 7. An example of equivalent query-graph.
Quolification Join-graph
(R1.A=R2.A)AND(R1.B=R3.B)
AND (R2.C=R3.C)
(a)

R1.A——R2.A
R1.B——R3.B
R2.C——R3.C
(b)

The relations R1, R2 and R3 cannot be fully reduced by semijoins

(c)

| RI |  | R3 |  | R2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | 8 | C |  | A |
| 0 | 2 | 2 | 4 | 4 | 1 |
| 1 | 3 | 3 | 5 | 5 | 0 |

(d)

Figure 8. An illustration that the relations in a query having a cyclic querygraph cannot be fully reduced by semijoins.
inadequate to fully reduce a cyclic query [Bernstein and Chiu 1981; Bernstein and Goodman 1981]. This is illustrated by the following example. The relations $R_{1}, R_{2}$, and $R_{3}$ are referred to by the query as given in Figure 8d. If the semijoin $R_{1}-A \rightarrow R_{2}$ is used, then $R_{2}$ remains unchanged because $R_{1} . A$ and $R_{2} . A$ are identical. Similarly, the semijoins $R_{2}-C \rightarrow R_{3}$ and $R_{3}-B \rightarrow R_{1}$ have no effect on $R_{3}$ and $R_{1}$, respectively. However, the fully reduced $R_{1}, R_{2}$, and $R_{3}$ should be the null relations, because no tuple of $R_{1}$, of $R_{2}$, and of $R_{3}$ simultaneously satisfies the qualification.

### 4.2 Tree Query Recognition Algorithm

Section 4.1 illustrates the importance of recognizing qualifications that have either tree query-graphs or are equivalent to other qualifications having tree query-graphs. It turns out that there is a simple algorithm [Graham 1979; Yu and Ozsoyoglu 1979] for the recognition of such queries, as follows. The algorithm takes a query as input, and it has two key steps. Initially, for each relation $R_{t}$, the set of attributes of the relation appearing in the qualification, $J\left(R_{t}\right)$, is constructed. As described earlier,
each attribute $A$ of $R_{t}$ in the qualification denotes a relationship between $R_{t}$ and the set of relations containing $A$.

In the first step, $R_{i}$ is eliminated from consideration, if a pair of relations $R_{t}$ and $R_{j}$ exists such that $J\left(R_{i}\right) \subseteq J\left(R_{j}\right)$. Condition $J\left(R_{\mathrm{r}}\right) \subseteq J\left(R_{j}\right)$ guarantees that an equivalent qualification can be constructed by substituting each clause of the form $R_{2} . X=R_{k} . X$, where $k \neq j$, with two clauses ( $R_{v} \cdot X=R_{j} . X$ ) AND ( $R_{j}: X=R_{k} \cdot X$ ). (This substitution may produce some duplicated clauses.) After substitution, $R_{t}$ appears only in the clause $R_{i} . X=R_{J} . X$. It is then clear that in the query-graph, $R_{t}$ is only adjacent to $R_{j}$, and therefore is not part of any cycle. Hence the elimination of $R_{t}$ will not change the type of the query-graph. In Figure 7, $J\left(R_{3}\right)=\left\{A_{2}\right\} \subseteq J\left(R_{1}\right)$. The edge between $R_{3}$ and $R_{4}$ is replaced by that between $R_{1}$ and $R_{4}$ (and that between $R_{1}$ and $R_{3}$ ). As a result, $R_{3}$ is not part of any cycle and can be eliminated without affecting the type of the query.

In the second step, if any relation is eliminated in Step 1, it is checked to determine whether it causes the elimination of an attribute. An attribute is to be eliminated if only one relation remains containing that attribute. (One should here recall that if a set of relations contains the same attribute, they are related by the equality of that attribute; thus if there is no more than one relation having that attribute, no such relationship exists.) For example, if $R_{t} . A=R_{j} . A$ is the only clause involving attribute $A$, and if $R_{t}$ is eliminated in Step 1 , then attribute $A$ will be eliminated in this step. It is clear that the elimination of an attribute causes the updating of the relation $R$ (more precisely, $J(R)$ ) having that attribute originally. The algorithm is simply an iteration of Steps 1 and 2. If all relations are eliminated at the end of the algorithm, the original query is then a tree query, because the algorithm does not affect the type of a query (tree or cyclic) and a null query is clearly a tree query. If some relations do exist at the end of the algorithm, then it can be shown that the original query is a cyclic query [ Yu and Ozsoyoglu 1979]. Figure 9 illustrates the

$$
\begin{aligned}
& J\left(R_{1}\right)=\left\{A_{1}, A_{2}\right\} \\
& J\left(R_{2}\right)=\left\{A_{1}\right\} \\
& J\left(R_{3}\right)=\left\{A_{2}\right\} \\
& J\left(R_{4}\right)=\left\{A_{1}, A_{2}\right\}
\end{aligned}
$$

Since $J\left(R_{2}\right) \subseteq J\left(R_{1}\right)$, eliminate $R_{2}$. $J\left(R_{3}\right) \subseteq J\left(R_{1}\right)$, eliminate $R_{3}$. $J\left(R_{4}\right) \subseteq J\left(R_{1}\right)$, eliminate $R_{4}$.

Since $A_{1}$ occurs in $R_{1}$ only, eliminate $A_{1}$. $A_{2}$ occurs in $R_{1}$ only, eliminate $A_{2}$.
$R_{1}$ does not have any attribute, eliminate $R_{1}$.
All relations are eliminated. Thus, this is a tree query.
Figure 9. Demonstrating that the query in Figure 7a is a tree query.
operation of the algorithm on the query given in Figure 7a.

A further characterization of cyclic queries should illustrate the concept more clearly [Goodman and Shmueli 1983]. There are two basic forms of cyclic queries as shown in Figure 10.

It is clear that the two operations used to determine a tree query will not eliminate any attribute or relation from the above query-graphs, and therefore $A$ ring and Aclique are cyclic queries.

All other cyclic queries can be reduced to either an Aring or an Aclique by repeated applications of the two operations and by eliminating a common set of attributes from each of the relations. In essence, the absence of A ring and Aclique implies that the query is a tree query. Numerous other characterizations of cyclic and tree queries are given in Beeri et al. [1981, 1983], Fagin et al. [1980], and Goodman and Shmueli [1983] relating distributed query processing to dependency theory, database schema design, and graph theory; these processes are not covered in this paper.

### 4.3 Transforming a Cyclic Query into a Tree Query

Since the tree query is fully reducible, algorithms capable of transforming a cyclic query to a tree query are desirable [Goodman and Shmueli 1982b; Kambayashi and Yoshikawa 1983; Kambayashi et al. 1982].

(b)

Figure 10. (a) $A$ ring of $n$-relationships; (b) $A$ clique of $n$-relationships.

Basically, there are three different transformation algorithms: (1) a relation-merging algorithm [Goodman and Shmueli 1982a; Kambayashi and Yoshikawa 1983], (2) a tuplewise decomposition algorithm, and (3) an attribute addition algorithm [Kambayashi et al. 1982]. Some details of the algorithms are as follows:
(1) The relation-merging algorithm simply joins certain relations residing in a cycle to eliminate the cycle. For example, given a cyclic query as shown in Figure 11a, the algorithm can join any two relations in the cycle. This causes the cycle to disappear. Figure 11b shows the query-graph resulting by joining $R_{2}$ and $R_{3}$ together.
(2) The tuple-wise decomposition algorithm is based on the tuple-substitution idea of Wong and Youssefi [1976]. The algorithm eliminates a cycle by decomposing a cyclic query into a number of tree subqueries. By first arbitrarily selecting a relation in a cycle, it constructs a tree subquery for each tuple of the relation by substituting the attribute values of the tuple. Using the query in Figure 11a, if relation $R_{3}$ is selected, $\left|R_{3}\right|$ subqueries will be generated, with each subquery corresponding to a tuple of $R_{3}$. Each subquery has a query graph as shown in Figure 11c. The answer of the query is then the union of all the answers of the subqueries.
(3) The attribute addition algorithm aims at fully reducing a relation in a cyclic
query. In Kambayashi et al. [1982], certain attributes of some relations involved in cycles are added to other relations in such a way that a tree query results. Semijoins can be then used to fully reduce any given relation. The algorithm takes as input a cyclic query and a relation to be fully reduced, for example, $R$. It then chooses a spanning tree from the query-graph of the query with $R$ as the root. Given the same example in Figure 11a, suppose that $R_{1}$ is to be fully reduced. Figure 11d-f gives the three possible spanning trees (the solid edges in each figure) with $R_{1}$ as the root. Since the query is cyclic, at least one edge is not in the spanning tree. For each edge not in the spanning tree (the dotted edge in each figure), the label of the edge is added to those of the edges that form a cycle with the edge. For example, in Figure 11d, $R_{3}-$ $B-R_{1}$ is the edge not in the spanning tree. Its label $B$ is added to the labels of the edges $R_{1}-A-R_{2}$ and $R_{2}-C-R_{3}$ to form the new edges $R_{1}-A B-R_{2}$ and $R_{2}$ $-B C-R_{3}$. (At this point, $R_{2}$, which originally does not have attribute $B$, is assumed to have all distinct values of $B$ so that semijoins on attribute $B$ involving $R_{2}$ will not produce null relations.) After this process, Figure 11d becomes a tree query. By the "leaves to root" algorithm given in Bernstein and Chiu [1981], $R_{1}$ can be fully reduced if the semijoins $R_{3}-B C \rightarrow R_{2}$ and $R_{2}-A B \rightarrow R_{1}$ are executed. Notice that if attribute addition had not been used, the

$0=\{R 1 . D \mid(R 1 . A=R 2 . A)$
$R 1-A B-R 23$
AND (R2.C=R3.C)
$A N D(R 1 . B=R 3 . B)\}$
where R23 = R2 natural join R3
(a)
(b)

(c)

semi-joins:
$R 2-A C \rightarrow R 1$,
$R 3-B C \rightarrow R 1$
(e)

semi-joins: $R 3-B C \rightarrow R 2$, $R 2-A B \rightarrow R 1$
(d)

semi-joins.
$R 2-A C \rightarrow R 3$,
$R 3-A B \rightarrow R 1$
(f)

Figure 11. Transforming cyclic query into tree query.

$D \Rightarrow B$ in R3
(a)
$D \Rightarrow B$ in R1,R3
(b)

(c)

Figure 12. Functional dependency and query graph.
corresponding semijoin sequence would have been $R_{3}-C \rightarrow R_{2}, R_{2}-A \rightarrow R_{1}$, and $R_{1}$ might not have been fully reduced.

Kambayashi and Yashikawa [1983] have studied the effect of functional and multivalued dependencies on query processing, and have identified a sufficient condition for a relation in a cyclic query to be fully reduced. Given a cyclic query as shown in Figure $12 \mathrm{a}, D \Rightarrow B$ can be considered a functional dependency identified in $R_{3}$. By executing a semijoin $R_{3}-B D \rightarrow R_{1}$, the resulting relation $R_{1}$ will preserve the functional dependency $D \Rightarrow B$. Thus in the current database, the values of attribute $B$ are uniquely determined if those of attribute $D$ are known. Consequently, attribute $B$ can be deleted, and the query-graph is changed to Figure 12b. Since all three relations contain attribute $D$, the querygraph is further simplified to Figure 12c, which is a tree. A relation in a cyclic query can therefore be fully reduced by semijoins if, "for each cycle in the query-graph,
(1) all relations in the cycle contain the attribute(s) $X$,
(2) an edge in the cycle is labeled $X Y$, and
(3) a relation on the edge has the functional dependency $X \Rightarrow Y$."

## 5. OPTIMAL STRATEGY FOR SIMPLE QUERIES

In the reduction phase, the query-processing algorithms described in the previous section are all heuristics and may not always yield optimal strategies. In this section we present an optimal algorithm for simple queries (all relations appearing in the qualification of such a query have the same attribute, and each relation has a single attribute). The discussion will proceed under the following assumptions. Let the common attribute be $A$. Let the relations be $\left\{R_{1}, R_{2}, \ldots, R_{n}\right\}$ situated at different sites. Let $R_{s}$ be the result site where the answer is to be produced.

As noted in Section 2, a strategy can be represented by a directed graph where the vertices are the relations and the edges represent the semijoins. It is clear that a
strategy should have all paths directed toward the result site $R_{s}$ and that the strategy should contain the $n$ relations, some of which may appear more than once. The cost of a strategy is the sum of the data transmission cost of executing the semijoins represented by the edges. Since it is impossible to find the precise cost of a strategy before its execution, the usual procedure is to minimize the expected cost. As an example, if a strategy is $R_{t 1}-A \rightarrow R_{t 2}$ $\cdots-A \rightarrow R_{t m}, m=n, R_{i m}=R_{s}$ and if $p_{t J}$ is the selectivity of relation $R_{i j}$ on attribute $A$, then the expected transmission cost of the strategy is $|A|\left[p_{i 1}+p_{i 1} p_{i 2}+\cdots+\right.$ $\left.p_{11} p_{22} \cdots p_{t m-1}\right]$. An optimal strategy is one having the smallest expected cost among the directed graphs satisfying the conditions noted above.

Some properties are satisfied by an optimal strategy for a simple query. They are listed as follows:

Property 5.0. All relations should appear on one directed path; that is, an optimal strategy for a simple query is a "string" of directed edges of the form $R_{t 1} \rightarrow R_{t 2} \rightarrow$ $R_{t 3} \rightarrow \cdots \rightarrow R_{t t}$.

In Hevner and Yao [1979], two cases are considered. In the first case, the result site is one of the sites containing $R_{1}, R_{2}, \ldots$, $R_{n}$. In the other case, the site does not contain any of the $n$ relations.

If the result site does not contain any of the $n$ relations, then all of the relations in an optimal strategy $R_{i 1} \rightarrow R_{i 2} \rightarrow \cdots \rightarrow R_{t t}$ are distinct; that is, no relation appears more than once. This conclusion is rather obvious, because if a relation occurs twice or more, then the second and subsequent occurrences of the relation can be removed from the strategy, yielding an equivalent but lower cost strategy. They are equivalent because the last relation in both strategies satisfies $R_{1} . A=R_{2} . A=\cdots=R_{n} . A$; the latter strategy has a lower cost because the second and subsequent occurrences of the relation, having appeared earlier, will not provide extra reduction to later relations.

Property 5.1 [Hevner and Yao 1979]. All relations should appear in ascending order of size.

Thus, the optimal strategy is in fact $R_{1} \rightarrow R_{2} \rightarrow \cdots \rightarrow R_{n} \rightarrow R_{s}$, where $\left|R_{1}\right| \leq$ $\left|R_{2}\right| \leq \cdots \leq\left|R_{n}\right|$.

One can verify the result by a straightforward check to see that if $R_{i 1} \rightarrow R_{i 2} \cdots$ $\rightarrow R_{y-1} \rightarrow R_{y j} \rightarrow R_{i j+1} \rightarrow R_{i j+2} \rightarrow \cdots \rightarrow R_{t t}$ is a strategy with $\left|R_{y}\right|>\left|R_{y+1}\right|$, then the strategy $R_{t 1} \rightarrow R_{\mathrm{t} 2} \rightarrow \cdots \rightarrow R_{y-1} \rightarrow$ $R_{i j+1} \rightarrow R_{y} \rightarrow R_{y+2} \rightarrow \cdots \rightarrow R_{i t}$ yields a lower cost.
If the result site $R_{s}$ contains a relation $R_{l}$, then the optimal strategy [Hevner and Yao 1979] is either $R_{1} \rightarrow R_{2} \rightarrow \cdots \rightarrow$ $R_{t-1} \rightarrow R_{l} \rightarrow R_{l+1} \rightarrow \cdots \rightarrow R_{s}$ or $R_{1} \rightarrow$ $R_{2} \rightarrow \cdots \rightarrow R_{t-1} \rightarrow R_{i+1} \rightarrow \cdots \rightarrow R_{s}$, where the former strategy passes through the result site twice and the latter strategy passes through the result site once. The lower cost strategy between the two strategies is taken as the optimal one. Optimal strategies can thus be easily obtained for simple queries.

## 6. OPTIMAL STRATEGY FOR TREE QUERIES

In this section, we provide an outline of a method to obtain optimal strategies to fully reduce a relation for tree queries with the restriction that any two relations have at most one single common joining attribute. The method will be illustrated by a special type of tree query in which there are $m$ single attribute relations, each having the joining attribute $A$; these are labeled $A_{1}$, $A_{2}, \ldots, A_{m}$ with $\left|A_{1}\right| \leq\left|A_{2}\right| \leq \cdots \leq$ $\left|A_{m}\right|$. There are $n$ single attribute relations, each having the joining attribute $B$, which are labeled $B_{1}, B_{2}, \ldots, B_{n}$ with $\left|B_{1}\right| \leq\left|B_{2}\right| \leq \cdots \leq\left|B_{n}\right|$, and a twoattribute relation $I$ having the joining attributes $A$ and $B$. We are given a query referring to the above $m A$ 's, $n B$ 's, and the $I$ relation. $\operatorname{OPTS}(m, n, I)$ is an optimal strategy to fully reduce some relation $Y$ in the query, where $Y$ should be the last relation in the sequence, and $\operatorname{OPTS}(m, n$, $I, X$ ) an optimal strategy to fully reduce a specific relation $X$ in the query.

Some properties of $\operatorname{OPTS}(m, n, I)$ are given as follows [Chen and Li 1983; Yu et al. 1979, 1982a]:
Property 6.0. Relations having a common attribute should all appear on one
directed path. (This is the generalized version of Property 5.0 for arbitrary queries.)

## Example 6.1


does not violate Property 6.0.

$$
\text { (b) } A_{1} \rightarrow A_{2} \rightarrow I \rightarrow B_{1} \rightarrow B_{2} \rightarrow B_{3} \rightarrow I
$$

is a possible optimal strategy.
(c) $A_{1} \rightarrow A_{2} \rightarrow I$

cannot be an optimal strategy, because $B_{1}$ and $B_{2}$ are not on the same directed path. This violates Property 6.0.
(d)

does not violate Property 6.0.

$B_{1}$ is a possible optimal strategy.

$$
\text { (f) } A_{1} \rightarrow I \rightarrow \underset{B_{1}}{A_{2} \rightarrow I \rightarrow B_{2} \rightarrow B_{3}}
$$

violates Property 6.0 because the first occurrence of $I$ and $B_{1}$ are in different paths.

Property 6.1. All $A$ 's and $B$ 's appear exactly once, while $I$ may appear once or more.

Example 6.2. The strategy in Example 6.1(d) violates Property 6.1 since $B_{1}$ appears twice in the strategy.
Property 6.2. Single-attribute relations must appear in ascending order of their sizes in the path leading to the first fully reduced relation $Y$.

Example 6.3. The strategy in Example 6.1(a) violates Property 6.2, because $A_{1}$ and $A_{2}$ are in descending order of size; the strat-
egies in Example 6.1(b) and 6.1(e) satisfy Property 6.2.

By Property 6.2, the first fully reduced relation in OPTS $(m, n, I)$ is $A_{m}$ or $B_{n}$ or $I$. Thus OPTS $(m, n, I)$ has one of the following three forms:

$$
\begin{aligned}
& \operatorname{OPTS}\left(m, n, I, A_{m}\right), \\
& \operatorname{OPTS}\left(m, n, I, B_{n}\right), \\
& \operatorname{OPTS}(m, n, I, I) .
\end{aligned}
$$

Consider $\operatorname{OPTS}\left(m, n, I, A_{m}\right)$. The vertex immediately preceding $A_{m}$ cannot be a $B$ relation since if a $B$-relation is sent to $A_{m}$, the sending is a waste because it cannot merge with $A_{m}$ directly. This vertex therefore must be either $I$ or $A_{m-1}$, by Property 6.2.

Subcase 1. If the vertex is $A_{m-1}$, the set of relations preceding $A_{m}$ then forms a substrategy involving the $m-1 A$-relations $\left\{A_{1}, A_{2}, \ldots, A_{m-1}\right\}$, the $n B$-relations $\left\{B_{1}\right.$, $\left.B_{2}, \ldots, B_{n}\right\}$, and the $I$-relation. This substrategy denoted by OPTS $(m-1, n, I$, $A_{m-1}$ ) is optimal among all substrategies ending at $A_{m-1}$ and involving the same subset of relations. (Otherwise, a better substrategy followed by the data transfer of the reduced $A_{m-1}$ to $A_{m}$ will produce a better strategy.)

Subcase 2. If the vertex is $I$, then again we have an optimal substrategy involving the same subset of relations. This substrategy is denoted by $\operatorname{OPTS}(m-1, n$, $I, I$ ), since the last vertex in the substrategy is $I$.

Both substrategies process the same set of relations, and the relation immediately following each of these substrategies is $A_{m}$. The amount of data thus transmitted from each of those substrategies to $A_{m}$ is identical and can be denoted by $Z$. OPTS $(m, n$, $I, A_{m}$ ) is either $A_{m}$ preceded by OPTS $(m-$ $1, n, I, I)$ or $A_{m}$ preceded by $\operatorname{OPTS}(m-1$, $n, I, A_{m-1}$ ). If $C$ (strategy) is the cost of the strategy, then

$$
\begin{aligned}
& C\left(\operatorname{OPTS}\left(m, n, I, A_{m}\right)\right) \\
& =\left\{c_{0}+c_{1} * Z\right)+\min \left\{C\left(\operatorname{OPTS}\left(m-1, n, I, A_{m-1}\right)\right),\right. \\
& \\
& C(\operatorname{OPTS}(m-1, n, I, I))\} .
\end{aligned}
$$

Pictorially, OPTS ( $m, n, I, A_{m}$ ) can be represented by

$$
\begin{align*}
& A_{m} \leftarrow \min \{\operatorname{OPTS}(m-1, n, I, I), \\
& \left.\quad \operatorname{OPTS}\left(m-1, n, I, A_{m-1}\right)\right\}, \tag{6.1}
\end{align*}
$$

where the cost functions are not explicitly written. Similarly, $\operatorname{OPTS}\left(m, n, I, B_{n}\right)$ is

$$
\begin{align*}
& B_{n} \leftarrow \min \{\operatorname{OPTS}(m, n-1, I, I), \\
& \left.\quad \operatorname{OPTS}\left(m, n-1, I, B_{n-1}\right)\right\} . \tag{6.2}
\end{align*}
$$

If the first fully reduced relation $I$ of $\operatorname{OPTS}(m, n, I, I)$ has in-degree 1 , then the relation immediately preceding $I$ can be either $A_{m}$ or $B_{n}$. The two subcases are, respectively,

$$
\begin{align*}
& I \leftarrow A_{m} \leftarrow \min \{\operatorname{OPTS}(m-1, n, I, I), \\
&  \tag{6.3}\\
& \left.\quad \operatorname{OPTS}\left(m-1, n, I, A_{m-1}\right)\right\}, \\
& I \leftarrow B_{n} \leftarrow \min \{\operatorname{OPTS}(m, n-1, I, I),  \tag{6.4}\\
& \\
& \left.\quad \operatorname{OPTS}\left(m, n-1, I, B_{n-1}\right)\right\} .
\end{align*}
$$

If the first fully reduced relation $I$ has indegree 2, then by Property 6.0 the optimal strategy is

where $I A$ and $I B$ are the projections of $I$ on the attributes $A$ and $B$, respectively.

OPTS $(m, n, I)$ is the minimal cost strategy among the five strategies given by (6.1)-(6.5) (see Example 6.4 for the end cases). It is clear from the equations above that $\operatorname{OPTS}(m, n, I, Y)$, where $Y=A_{m}, B_{n}$, or $I$, can be computed in constant time if $\operatorname{OPTS}(m-1, n, I, I), \operatorname{OPTS}(m-1, n, I$, $\left.A_{m-1}\right), \operatorname{OPTS}(m, n-1, I, I), \operatorname{OPTS}(m, n$ $\left.-1, I, B_{n-1}\right), \operatorname{OPTS}\left(m, 0,0, A_{m}\right), \operatorname{OPTS}(m$, $\left.0, I A, A_{m}\right), \operatorname{OPTS}\left(0, n, I B, B_{n}\right)$, and $\operatorname{OPTS}\left(0, n, 0, B_{n}\right)$ are known. The following method is suggested to obtain the optimal strategy.

In the two-dimensional figure in Figure 13, the point ( $i, j$ ) denotes three optimal strategies involving $\left\{A_{1}, \ldots, A_{i}, B_{1}, \ldots\right.$, $\left.B_{j}, I\right\}$, one ending in $A_{i}$, one ending in $B_{j}$,


Figure 13. Illustration of how the optimal strategy is obtained.
and one ending in $I$. From eqs. (6.1)-(6.5), the optimal strategies at $(m, n)$ can be obtained from those at ( $m-1, n$ ), ( $m$, $n-1),(m, 0)$, and $(0, n)$. If we compute all optimal strategies at $\left(x_{1}, x_{2}\right), x_{1}+$ $x_{2}=t$, and at the boundary points ( $\left.i, 0\right)$, $(0, j), 1 \leq i \leq n, 1 \leq j \leq m$ (the optimal strategies at the boundary points involving essentially single-attribute relations are easily computable [Hevner and Yao 1979]), then the strategies at ( $y_{1}, y_{2}$ ), $y_{1}+$ $y_{2}=t+1$ can easily be computed using (6.1)-(6.5). Starting from $t=2$, we progress to $t=m+n$ when the optimal strategy for the query is obtained. This operation can be shown to take $O(m n)$ time [ Yu et al. 1979].

Example 6.4. Given $\left|A_{1}\right| \leq\left|A_{2}\right| \leq$ $|I A|,|I B| \leq\left|B_{1}\right|$, and $m=2, n=1$; see Figure 14.
(1) For the points on the $A$ axis, the optimal strategies $\operatorname{OPTS}\left(k, 0,0, A_{k}\right)$, $\operatorname{OPTS}\left(k, 0, I A, A_{k}\right)$, and $\operatorname{OPTS}(k, 0, I, I)$, where $k=1$ or 2 , are calculated as follows:
$\operatorname{OPTS}\left(k, 0,0, A_{k}\right):$

$$
\begin{array}{ll}
A_{1}, & k=1, \\
A_{1} \rightarrow A_{2}, & k=2
\end{array}
$$

$\operatorname{OPTS}\left(k, 0, I A, A_{k}\right):$

$$
\begin{aligned}
\min \left\{A_{1}\right. & \left.\rightarrow I A \rightarrow A_{1}, I A \rightarrow A_{1}\right\}, \quad k=1, \\
\min \left\{A_{1}\right. & \rightarrow A_{2} \rightarrow I A \rightarrow A_{2}, \\
A_{1} & \left.\rightarrow I A \rightarrow A_{2}\right\}, \quad k=2 .
\end{aligned}
$$

$\operatorname{OPTS}(k, 0, I, I)$ :

$$
\begin{array}{ll}
A_{1} \rightarrow I, & k=1, \\
A_{1} \rightarrow A_{2} \rightarrow I, & k=2 .
\end{array}
$$

(2) For the points on the $B$ axis, the optimal strategies $\operatorname{OPTS}\left(0,1,0, B_{1}\right)$, $\operatorname{OPTS}\left(0,1, I B, B_{1}\right)$, and $\operatorname{OPTS}(0,1, I, I)$ are calculated as follows:
$\operatorname{OPTS}\left(0,1,0, B_{1}\right): \quad B_{1}$,
$\operatorname{OPTS}\left(0,1, I B, B_{1}\right): \quad I B \rightarrow B_{1}$,
$\operatorname{OPTS}(0,1, I, I)$ :
$\min \left\{I B \rightarrow B_{1} \rightarrow I, B_{1} \rightarrow I\right\}$.


Figure 14. Example 6.4.
Then, all points ( $x_{1}, x_{2}$ ) satisfying $x_{1} \geq 1$, $x_{1}+x_{2}=2$ are located. In our example, $(1,1)$ is the only point. The three optimal strategies to be considered are $\operatorname{OPTS}(1,1$, $\left.I, A_{1}\right), \operatorname{OPTS}(1,1, I, I)$, and $\operatorname{OPTS}(1,1$, $I, B_{1}$ ).

By (6.1) and 6.2),
$\operatorname{OPTS}\left(1,1, I, A_{1}\right)$

$$
\text { is } \quad A_{1} \leftarrow \operatorname{OPTS}(0,1, I, I)
$$

and
$\operatorname{OPTS}\left(1,1, I, B_{1}\right)$

$$
\text { is } \quad B_{1} \leftarrow \operatorname{OPTS}(1,0, I, I)
$$

By (6.3), and (6.4) and (6.5), $\operatorname{OPTS}(1,1$, $I, I)$ is the minimal cost strategy among the following three strategies:

$$
\begin{aligned}
& I \leftarrow \operatorname{OPTS}\left(1,1, I, A_{1}\right), \\
& I \leftarrow \operatorname{OPTS}\left(1,1, I, B_{1}\right),
\end{aligned}
$$

and

$$
\begin{array}{r}
\min \left\{\operatorname{OPTS}\left(1,0,0, A_{1}\right),\right. \\
\left.\operatorname{OPTS}\left(1,0, I, A_{1}\right)\right\}, \\
\min \left\{\operatorname{OPTS}\left(0,1,0, B_{1}\right),\right. \\
\left.\operatorname{OPTS}\left(0,1, I, B_{1}\right)\right\} .
\end{array}
$$

There is only one point $(2,1)$ satisfying $x_{2} \geq 1, x_{1}+x_{2}=3$. The three optimal strategies at $(2,1)$ are calculated by (6.1)(6.5). They are as follows:
$\operatorname{OPTS}\left(2,1, I, A_{2}\right)$ :
$A_{2} \leftarrow \min \{\operatorname{OPTS}(1,1, I, I)$,

$$
\left.\operatorname{OPTS}\left(1,1, I, A_{1}\right)\right\}
$$

$\operatorname{OPTS}\left(2,1, I, B_{1}\right):$
$B_{1} \leftarrow \operatorname{OPTS}(2,0, I, I)$,
and
$\operatorname{OPTS}(2,1, I, I)$ :

$$
\begin{aligned}
& \min \{I \leftarrow \operatorname{OPTS}\left(2,1, I, A_{2}\right), \\
& I \leftarrow \operatorname{OPTS}\left(2,1, I, B_{1}\right), \\
& \min \left\{\operatorname{OPTS}\left(2,0,0, A_{2}\right),\right. \\
& I^{\operatorname{Lin}}\left.\operatorname{OPTS}\left(2,0, I A, A_{2}\right)\right\} \\
& \min \{ \operatorname{OPTS}\left(0,1,0, B_{1}\right), \\
&\left.\operatorname{OPTS}\left(0,1, I B, B_{1}\right)\right\} .
\end{aligned}
$$

If the minimum cost strategy to fully reduce some relation is sought, the answer is then

$$
\begin{aligned}
& \min \left\{\operatorname{OPT}\left(2,1, I, A_{2}\right), \operatorname{OPT}\left(2,1, I, B_{1}\right)\right. \\
& \operatorname{OPT}(2,1, I, I)\} .
\end{aligned}
$$

The algorithm can be generalized to obtain optimal strategies to fully reduce a relation for tree queries (see Chiu and Ho [1980] and Yu et al. [1979]). However, the algorithm runs in exponential time.

## 7. HEURISTICS ALGORITHMS BASED ON SEMIJOINS

Two query-processing algorithms using semijoins are discussed in this section. They assume that one copy of each relation referred to by the query has been selected and then the reduction and the assembly phases are carried out. The cost of a semijoin $X-A \rightarrow Y$ is defined to be the cost of transferring $X . A$ from the site containing $X$ to the site containing $Y$ (if the two sites are identical, the cost is zero). The benefit of the semijoin is the size of $Y$ before the operation minus the size of $Y$ after the operation. A semijoin is profitable if its cost is less than its benefit.

### 7.1 The SDD-1 Query-Processing Algorithm and Its Enhancements [Bernstein et al. 1981; Goodman et al. 1979]

The reduction phase is very simple; it identifies all possible semijoins between any two relations. The cost and the benefit of each semijoin are estimated. A profitable semijoin having the smallest cost is then chosen. (In one of the two papers, the semijoin
having the highest (benefit - cost) is selected.) The costs and the benefits of those semijoins that can be affected by the execution of the chosen semijoin are updated, and another semijoin is considered. The process is repeated until no profitable semijoin can be found. Some details are as follows.

First, all local reductions using selections and projections are performed. Semijoins within the same site can also be executed to reduce the sizes of relations. Then all possible semijoins across sites are identified. As pointed out earlier, after renaming attributes, all semijoins are of the form $R_{t} \rightarrow A_{k} \rightarrow R_{j}$, because semijoins of the form $R_{r} . A_{k} \rightarrow R_{j} . A_{l}$ are neither stated nor implied by the qualification of the query. For each such semijoin across sites, the cost and the benefit are estimated to be $p_{i}\left|A_{k}\right| w$ and $|R| \mid w_{j}\left(1-p_{i}\right)$, respectively, where $p_{\imath}$ is the selectivity of $R_{2}$ on attribute $A_{k},\left|A_{k}\right|$ is the cardinality of $A_{k}, w$ is the average width of a value in $A_{k}, w_{j}$ is the average width of a tuple in relation $R_{J}$, and $\left|R_{J}\right|$ is the number of tuples of the relation. The semijoin is profitable if $p_{i}\left|A_{k}\right| w<$ $\left|R_{J}\right| w_{j}\left(1-p_{i}\right)$. After identifying all profitable semijoins, the semijoin with least cost is selected to be the first semijoin to be executed, for example, $R_{r}-A_{s} \rightarrow R_{t}$. (The second version of SDD-1 selects the semijoin which maximizes (benefit - cost).) This semijoin is not executed until the entire sequence of semijoins and the assembly site are chosen. In spite of not executing this semijoin immediately, its effect on the relation $R_{t}$ is estimated. Specifically, the benefits and the costs of semijoins from $R_{t}$ to other relations have to be updated, due to expected reduction of $R_{t}$. After the update is performed, the next semijoin to be executed is chosen with the same criterion: that is, in the first version, the semijoin that has the least cost and is still profitable; and in the second version, the semijoin that has the largest (benefit - cost) and is still profitable. This process is repeated until all possible profitable semijoins have been exhausted.

The assembly phase consists of selecting, among all the sites, the site to which the transmission of all the relations referred to by the query incurs the minimum cost. The
site is chosen to be the one containing the largest amount of data after the reduction phase so that the sum of the amount of data transferred from other sites will be minimum. After selecting the assembly site, it may be possible to discard some useless semijoins. If, for example, relation $R$ resides in the assembly site and $R$ is scheduled to be reduced by a semijoin, but is not used to reduce other relations after the scheduled execution of the semijoin, then since $R$ need not be moved to another site during the assembly phase, the semijoin on $R$ is useless and should therefore be discarded.

The operations generated by the SDD-1 algorithm can be improved in the following ways [Yu et al. 1983]. Certain relations that are involved in the execution of semijoins need not be sent to the assembly site for further processing, and therefore both communication cost and local processing cost are saved. Furthermore, semijoins involving these relations can either be eliminated or replaced by other semijoins, yielding a smaller communication cost. Some details are provided below.

When a semijoin, for example, $R_{\imath}$ $X \rightarrow R$, is executed, not only is $R$, reduced, but in some situations, the contents of $R_{z}$ are completely incorporated into the resulting $R_{j}$ so that $R_{i}$ will not be needed for processing of the query. More precisely, let $J\left(R_{t}\right)$ be the joining attributes of $R_{i}$. If (i) $X=J\left(R_{t}\right)\left(\right.$ which implies $\left.J\left(R_{t}\right) \subseteq J\left(R_{j}\right)\right)$ and (ii) the target of the query either does not contain any attribute of $R_{b}$ or is equivalent to one without any attribute of $R_{t}, R_{i}$ can be eliminated from further consideration after executing the semijoin. In Figure $15 \mathrm{a}, A_{k}$ is the only joining attribute of $R_{t}$ and the semijoin is $R_{t}-A_{k} \rightarrow R_{J}$. Thus if the semijoin executed is $R_{\mathfrak{r}}-A_{k} \rightarrow R_{j}$, Condition (i) is satisfied. If the target of the query is that given in Figure 15b, which does not contain $R_{i}$, or that given in Figure 15 c , which can be transformed to one not containing $R_{t}$, then $R_{t}$ can be eliminated after executing the semijoin. Note that Condition (i) is precisely one of the two key steps in determining whether a given query is a tree query. When it is satisfied, the part of the query containing $R_{i}$ is a tree (sub)query. In Figure 15a, $R_{r}-A_{k}-R_{j}$ is a tree subquery, which permits $R_{j}$ to be

(a)
$\begin{aligned} \text { Torget } & =(R t \cdot A r, R m \cdot A u, R i, A k) \\ & =(R t \cdot A r, R m \cdot A u, R j \cdot A k)\end{aligned}$
(c)

Target $=(R t . A r, R m . A u)$
(b)

Torget = (Ri.Ag, Rt.Ar, Am.Au)
(d)

Figure 15. Possıbility of eliminating $R_{t}$ after executing the semijoin $R_{t}$ $A_{k} \rightarrow R_{\text {, }}$ : (a) query graph; (b) target; (c) another target; (d) another target.
fully reduced by $R_{1}$ with respect to it. On the other hand, Condition (i) is not satisfied by semijoins involving any two of the three relations $\left\{R_{f}, R_{t}, R_{m}\right\}$ in Figure 15a, for an obvious reason: The subquery involving the three relations is cyclic. Figure 15d shows a target that cannot be transformed into an equivalent target without the relation $R_{l}$. Thus considering the target given in Figure 15d, $R_{r}$ cannot be eliminated even if Condition (i) is satisfied. The satisfaction of both Conditions (i) and (ii) allows $R_{t}$ to be eliminated after the execution of the semijoin.
As pointed out in the last paragraph, SDD-1 does not recognize that certain relations involved in previously executed semijoins are not needed for further processing and can be eliminated, and that semijoins involving these relations can still be generated subsequently. It turns out [ Yu et al. 1983] that any semijoin involving any such disposable relation can always be replaced by another semijoin such that the cost of the new strategy is not higher than that of the original strategy. The replacement procedure begins by letting the semijoin be replaced by $R_{t}-A \rightarrow R_{1}$, where $R_{t}$ or $R_{\text {, or both relations can be eliminated. }}$ $R_{t}$ will be replaced by $R_{r}$, where, if $R_{t}$ cannot be eliminated, $R_{r}$ is $R_{i}$; otherwise, there exists a sequence of semijoins such that $R_{v}$ $\rightarrow R_{t 1}$ causes $R_{i}$ to be eliminated, $R_{t j} \rightarrow$ $R_{v+1}$ causes $R_{u}$ to be eliminated, $1 \leq j \leq t$, $R_{t+1}$ is not eliminated, and $R_{r}$ is $R_{t+1}$. If the relation replacing $R_{\imath}$ is denoted by $\operatorname{Repl}\left(R_{\mathrm{v}}\right)$, the same replacement procedure
applies to $R_{J}$. In the example of Figure 16, no relation is eliminated initially; thus $\operatorname{Repl}\left(R_{t}\right)=R_{t}, 1 \leq i \leq 4$. After the first semijoin $R_{1}-C \rightarrow R_{2}, R_{1}$ is eliminated and therefore $\operatorname{Repl}\left(R_{1}\right)=R_{2}$, as shown in Figure 16b. If the next semijoin is $S_{1}: R_{1}-C \rightarrow$ $R_{4}$, then the replacement semijoin is $S_{2}$ : $R_{2}-C \rightarrow R_{4}$. Since $R_{1}$ was used to reduce $R_{2}$, it is clear that $R_{1}(C)$ in semijoin $S_{1}$ contains $R_{2}(C)$ in semijoin $S_{2}$, and therefore $\operatorname{cost}\left(S_{2}\right) \leq \operatorname{cost}\left(S_{1}\right)$. Furthermore, $R_{4}$, which is reduced by semijoin $S_{1}$, contains $R_{4}$, which is reduced by semijoin $S_{2}$. Any semijoin thus originating from the latter $R_{4}$ has a smaller cost than the corresponding semijoin originating from the former $R_{4}$, and if no further semijoin is executed on $R_{4}$, the cost to send the latter relation to the assembly site is smaller than that to send the former relation to the same destination.
Instead of having the next semijoin be $S_{1}$, the next semijoin is $S_{3}: R_{4}-C \rightarrow R_{1}$. It is then replaced by $S_{4}: R_{4}-C \rightarrow R_{2}$. Since $R_{1}$ is not needed for processing the query, the semijoin $S_{3}$ is not a useful operation. And since $R_{1}(C)$ after executing $S_{3}$ contains $R_{2}(C)$ after executing $S_{4}$, any semijoin originating from $R_{1}(C)$ will be more costly than the corresponding one from $R_{2}(C)$. Thus in both cases, replacing $R_{1}$ by $R_{2}$ yields a better strategy.

Figure 16c-e shows that after executing some other semijoins other relations are eliminated, and defines the relations that should be replaced by other particular relations at each stage.

$$
Q=\left\{R_{2} \cdot D \mid\left(R_{1} \cdot C=R_{2} \cdot C\right) \text { and }\left(R_{4} \cdot C=R_{1} \cdot C\right) \text { and }\left(R_{2} \cdot D=R_{3} \cdot D\right)\right\}
$$

Initially, no relation is eliminated.
(a)

After execution of the semijoin
$R_{1}-C \rightarrow R_{2}$,
$R_{1}$ is eliminated
$\operatorname{Repl}\left(R_{1}\right)=R_{2} ;$
$\operatorname{Repl}\left(R_{2}\right)=R_{2} ;$
$\operatorname{Repl}\left(R_{3}\right)=R_{3} ;$
$\operatorname{Repl}\left(R_{4}\right)=R_{4}$.

## After

$R_{2}-C \rightarrow R_{4}$,
there is no change, that is,
$\operatorname{Repl}\left(R_{1}\right)=R_{2} ;$
$\operatorname{Repl}\left(R_{2}\right)=R_{2} ;$
$\operatorname{Repl}\left(R_{3}\right)=R_{3} ;$
$\operatorname{Repl}\left(R_{4}\right)=R_{4}$.
(b)
(c)
$R$ and send the resulting $R$ to some other site. $\mathrm{BST}_{1}, \mathrm{BST}_{2}, \ldots, \mathrm{BST}_{q}$ are combined to form a strategy to reduce $R, 1 \leq q \leq p$. As $q$ varies from 1 to $p, p$ combined strategies are formed. The combined strategy having the smallest cost is the least cost strategy to reduce $R$. A similar strategy for each relation is produced at the reduction phase.
In the assembly phase, the reduced relations are then sent to the result site to produce the answer as in the following example.

Example 7.1. Suppose that $R_{1}, R_{2}, R_{3}$, and $R_{4}$ are four relations, each residing in a different site. Let $Q=\left\{\left(R_{3} . X\right.\right.$, $\left.R_{4} . Y\right) \mid\left(R_{1} . A=R_{2} . A\right)$ AND ( $\left.R_{2} \cdot A=R_{4} \cdot A\right)$ $\left.\operatorname{AND}\left(R_{3} \cdot B=R_{1} . B\right)\right\}$.
Figure 17a describes the size of the domain of each joining attribute, the size of each relation, and the selectivity of each relation on each joining attribute.
Figure 17b presents the candidate schedules for attributes $A$ and $B$. Assume that $c_{0}=0$ and $c_{1}=1$. The best strategies for relation $R_{1}$ on attributes $A$ and $B$ are then obtained. They are $\mathrm{BST}_{1}: R_{3}-B \rightarrow R_{1} \rightarrow$ and $\mathrm{BST}_{2}: R_{4}-A \rightarrow R_{1} \rightarrow$, where $\operatorname{cost}\left(\mathrm{BST}_{1}\right)=400$ and $\operatorname{cost}\left(\mathrm{BST}_{2}\right)=440$. The combined strategies to reduce $R_{1}$ are

$$
R_{3}-B \rightarrow R_{1} \rightarrow
$$

and


The least cost strategy to reduce $R_{1}$ is then selected from the above two combined strategies, and the least cost strategies of $R_{2}, R_{3}$, and $R_{4}$ are selected by a similar process. Figure 17d shows the least cost strategies for all four different relations.
Figure 17 e gives the final strategy to answer the query.

### 7.3 Better Semijoin Sequence

Each of the two algorithms above constructs a semijoin strategy to answer a
given query. However, each of the constructed semijoin strategies can sometimes be improved. In Luk and Luk [1980], a polynomial time algorithm is presented to transform a given semijoin strategy (produced by some heuristic) into an equivalent strategy such that each semijoin in the former strategy corresponds to a semijoin in the latter strategy and incurs neither higher cost nor lower benefit. (The enhancements in SDD-1 given in Section 7.1 are applicable when one or more relations are eliminated. Here, the procedure is applicable even if no relation is eliminated.)
For example, the node $R_{1}$ in substrategy $R_{2}-A B \rightarrow R_{1} \leftarrow B-R_{3}$ has in-degree $>1$, and the label in one semijoin is a subset of the label in the other. Satisfying the conditions above guarantees that a better strategy can be obtained. $R_{3}-B \rightarrow R_{2}-$ $A B \rightarrow R_{1}$ is an example of such a strategy, because although the semijoin $R_{3}-B \rightarrow$ $X$ ( $X$ is $R_{1}$ in the former strategy and is $R_{2}$ in the latter strategy) is executed with the same cost in both strategies, the semijoin $R_{2}-A B \rightarrow R_{1}$ is executed with a smaller cost in the latter strategy. This process can be applied to $R_{3}$, if it should satisfy the above conditions. The algorithm by Luk and Luk [1980] scans a given strategy and identifies the situations in which a node has either in-degree $>1$ or out-degree $>1$, and checks whether the label in one semijoin involving the node is a subset of that in another semijoin involving the same node. When such a situation is detected, the algorithm replaces the substrategy by a better one. This process is applied to the preceding nodes recursively until no such situation exists.

## 8. ALGORITHMS BASED ON JOINS

Although the use of semijoins reduces the amount of data transfer and is a valuable tool, it is not always superior to the use of joins only. One reason is that for certain networks, the number of messages exchanged rather than the amount of data transferred may be the dominating factor. Additional messages may be generated when semijoins are employed. Another reason is that local processing costs can be significant, and since SDD-1 and related

|  |  | Selectivity |  |  |
| :---: | ---: | :---: | :---: | :---: |
| Size | $A$ | $B$ |  |  |
| $R_{1}$ | 1200 | 0.2 | 0.5 |  |
| $R_{2}$ | 600 | 0.6 | 0.25 |  |
| $R_{3}$ | 1200 |  |  |  |
| $R_{4}$ | 2000 | 0.2 |  |  |

Cardinality of $A=1000$.
Cardinality of $B=400$.
$c_{0}=0, c_{1}=1$.
Each distinct value in $A$ and each distinct value in $B$ have unit length.

$$
Q=\left\{\left(R_{3} \cdot X, R_{4} \cdot Y\right) \mid\left(R_{1} \cdot A=R_{2} \cdot A\right) \quad \text { AND } \quad\left(R_{2} \cdot A=R_{4} \cdot A\right) \quad \text { AND } \quad\left(R_{1} \cdot B=R_{3} \cdot B\right)\right\}
$$

(a)

| $A:$ cost |  | B: cost |  |
| :---: | :---: | :---: | :---: |
| $200 \quad R_{1}$ | $R_{1}-A \rightarrow$ | 100 | $R_{3}-B \rightarrow$ |
| $200+40 \quad R$ | $R_{1}-A \rightarrow R_{4}-A \rightarrow$ | $100+50$ | $R_{3}-B \rightarrow R_{1}-B \rightarrow$ |
| $\begin{aligned} & 200+40 \quad R_{1}-A \rightarrow R_{4}-A \\ & +24 \end{aligned}$ |  |  |  |
| $R_{1}$ <br> cost | $A$ | cost | $B$ |
| $200+240$ | $240 \quad R_{4}-A \rightarrow R_{1}$ | $100+300$ | $R_{3}-B \rightarrow R_{1} \rightarrow$ |
| $\begin{aligned} & R_{2}: \\ & \quad 200+40 \\ & \quad+24 \end{aligned}$ | $R_{1}-A \rightarrow R_{4}$ |  |  |
| $R_{3}$ : |  | $\begin{gathered} 100+50 \\ +600 \end{gathered}$ | $R_{3}-B \rightarrow R_{1}-B \rightarrow R_{3} \rightarrow$ |
| $\begin{aligned} & R_{4}: \\ & \quad 200+40 \\ & +24+ \end{aligned}$ | $\begin{aligned} & 40 \\ & +240 \end{aligned} R_{1}-A \rightarrow R_{4}-$ | $A \rightarrow R_{4} \rightarrow$ |  |

(c)

(d)

$$
\operatorname{R}_{1}-A \rightarrow R_{4}-A \rightarrow R_{2} \rightarrow \underset{\uparrow}{R_{4}-A \rightarrow R_{1}}
$$

(e)

Figure 17. An example illustrating the general algorithm in Apers et al. [1983]. (a) Size of domain of each joining attribute, size of each relation, and selectivity of each relation on each joining attribute; (b) candidate schedules for each joining attribute; (c) best strategies for reducing each relation on each of its attributes; (d) least cost strategies of different relations; (e) strategy for answering the query.
algorithms ignore these costs, the actual processing cost of strategies based on these algorithms can be high. Last, although semijoins can be executed in parallel, the minimization of response time using semijoins is complicated [Apers et al. 1983]. Several algorithms using joins are studied below.

### 8.1 Enumerative Algorithms

### 8.1.1 Algorithm in Epstein

 and Stonebraker [1980]The algorithm first partitions the set of relations in the query into two complementary groups, $G_{1}$ and $G_{2}$, where $G_{1}$ has at least two relations and $G_{2}$ has zero or more relations. Substrategy for the relations in $G_{1}$ is next obtained by designating the site containing the largest relation as the result site and sending all other relations in $G_{1}$ to it. It seeks the minimal cost substrategy by a recursive call for the relations in $G_{2} \cup$ $\{R\}$, where $R$ is the resulting relation obtained from the those relations in $G_{1}$. All possible combinations of $G_{1}$ and $G_{2}$ are considered to obtain the minimal strategy.
Should relations $R_{1}, R_{2}$, and $R_{3}$ reside in different sites and a query asks for the join of these three relations, the algorithm will first partitions $R_{1}, R_{2}$, and $R_{3}$ into $\left\{\left\{R_{1}, R_{2}\right\}\right.$, $\left\{R_{3}\right\}$. Then the minimal cost substrategy for $\left\{R_{1}, R_{2}\right\}$ is constructed by sending the smaller of the two relations $R_{1}$ and $R_{2}$ to the other. The relation obtained by joining $R_{1}$ with $R_{2}$, for example, $T_{1}$, is added to the second group and the minimal cost substrategy for $\left\{T_{1}, R_{3}\right\}$ is sought. A strategy for the joins of $R_{1}, R_{2}$, and $R_{3}$ is then obtained. The same process is repeated for $\left\{\left\{R_{1}, R_{3}\right\},\left\{R_{2}\right\}\right\},\left\{\left\{R_{2}, R_{3}\right\},\left\{R_{1}\right\}\right.$, and $\left\{\left\{R_{1}, R_{2}, R_{3}\right\}\right.$, \{ \}\}. At the end, the optimal strategy for the query is obtained.
In general, when the natural join of $n$ relations is sought, the exhaustive enumerative search algorithm will scan through $e(n)$ strategies, where

$$
\begin{aligned}
& e(1)=1 \\
& e(2)=1 \\
& e(n)=\sum_{i=2}^{n}\binom{n}{i} * e(n-i+1)
\end{aligned}
$$

where $\binom{n}{2}$ stands for the number of different combinations for the first group having $i$ relations, and $e(n-i+1)$ stands for the number of substrategies that the recursive call will scan through if the first group has $i$ relations.

By leaving out the lower order terms (i.e., $i \geq 3$ ) in the above expression, $e(n) \geq$ $\binom{n}{2} e(n-1) . e(4)=29 ; e(5) \geq 10 * 29=290 ;$ $e(6) \geq 15 \times 290=435$. Thus $e(n)$ grows very rapidly, although some of the strategies are degenerate (i.e., certain subsets of relations may not be joined, but strategies involving these subsets of relations are enumerated).

### 8.1.2 $R^{*}$ [Williams et al. 1981]

As in Epstein and Stonebraker [1980], R* enumerates many strategies and chooses the one with the least cost. However, many more alternatives are considered in $\mathrm{R}^{*}$. If a relation is replicated, the choice of the appropriate copies of the relation to be used for processing the query has a significant effect on the cost; the sequence in which the operations are performed is also important. For example, the cost of the strategy ( $\left(R_{1}\right.$ joined with $R_{2}$ ) joined with $R_{3}$ ) differs from that of the strategy ( $R_{1}$ joined with ( $R_{2}$ joined with $R_{3}$ ). Even the join between two different relations $R_{1}$ and $R_{2}$, situated at distinct sites $S_{1}$ and $S_{2}$, respectively, can be performed in several ways, resulting in different costs, for example:
(i) Send $R_{1}$ to site $S_{2}$ and join with $R_{2}$ there.
(ii) Send $R_{2}$ to site $S_{1}$ and join with $R_{1}$ there.
(iii) Send both relations $R_{1}$ and $R_{2}$ to a different site $S_{3}$ and join them there.
(iv) For each tuple of $R_{1}$ transmitted to $S_{2}$, send the matching tuples of $R_{2}$ to $S_{1}$.
(v) The same as (iv), with the roles of $R_{1}$ and $R_{2}$ reversed.

Many strategies are thus evaluated by $\mathrm{R}^{*}$, which takes into consideration both local processing cost and data communication cost. Although enumerating all these strategies for a query can be costly, this approach can be worthwhile if the query is frequently executed. Such an approach is
also taken in centralized databases [Griffiths Selinger et al. 1979].

### 8.2 Nonenumerative Algorithms

### 8.2.1 Algorithm in Baldissera et al. [1979]

The algorithm in Baldissera et al. [1979] accepts only tree queries. It decomposes a query into chain queries and solves them to obtain the answer. A chain query is a query whose query-graph or equivalent query-graph is a chain. A nonchain query is a query for which none of its equivalent query graphs is a chain.
Suppose that a chain query with a node designated as root is given. The algorithm finds the assembly site, which is the site with the maximum number of data referenced by the query. Then the algorithm repeats the following process until the answer to the chain query is obtained. Starting from a leaf, the algorithm checks whether joining the leaf with its parent first and then sending the result to the assembly site incurs less cost than sending the two relations directly to the assembly site and performing the join there. If the former strategy is less costly, then the leaf node and its parent are merged to form a temporary relation, and the query graph is modified by replacing the part of the graph connecting the two relations by the newly created relation. Otherwise, the leaf node is sent to the assembly site and is eliminated from the query graph. This process is repeated over the modified query graph. When two relations are joined, the algorithm sends the smaller relation to the site containing the larger relation and merges them.
Figure 18c gives an example of a chain query. A decision has to be made whether to merge the leaf $R_{5}$ and its parent $R_{34}$ to form a new relation $R_{345}$ or to send $R_{5}$ and $R_{34}$ directly to the root. The choice with the lower transmission cost is selected.
If the query graph is a tree but not a chain, and $R$ is the root, adjacent to $k$ nodes, the following two cases arise:
Case 1. $k>1$. The tree is decomposed into $k$ subtrees, with each subtree containing $R$ as the root. $R$ is the only node in
common between the subtrees. For example, the subtrees of Figure 18a are given in Figure 18d. The subtree with the smallest number of nodes is first selected for processing. If the subtree is a chain, the previous procedure is applied to the subquery corresponding to the subtree, and the root is modified and incorporated into the original tree. For example, processing the chain in Figure 18d and incorporating the modified root into the original tree yields the modified query-graph in Figure 18b. If the selected subtree is not a chain, then the present procedure is applied recursively to the subtree.

Case 2. $k=1$. Let the direct descendant of $R$ be $r$. If $r$ has two or more direct descendants, then the subtree with root $r$ is identical to Case 1 and the same procedure is applied to it; otherwise, process in Case 2 is applied.

Example 8.1. Given a tree query with root $R_{1}$ as shown in Figure 18a, since $R_{1}$ is adjacent to two nodes, the tree is decomposed into two subtrees, as shown in Figure 18d. Let the corresponding subqueries be $Q_{1}$ and $Q_{2}$. Since $Q_{1}$ is a chain query, it is processed as described above. Relations $R_{1}$ and $R_{2}$ are merged to form the modified root $R_{12}$, which is incorporated into the original tree to form the modified query graph as shown in Figure 18b. This modified query graph has one subtree only, and thus it belongs to Case 2. The direct descendant of $R_{12}, R_{4}$, has two direct descendants. Thus the subtree with root $R_{4}$ is decomposed into two subtrees, namely $R_{4}$ -$C-R_{3}$ and $R_{4}-D-R_{5}$. Suppose that the former subtree is selected for processing. $R_{3}$ and $R_{4}$ are merged to form $R_{34}$, and the modified query-graph is that shown in Figure 18c. Since this is a chain query, the procedure for processing chain queries is invoked.

### 8.2.2 The INGRES Algorithm [Epstein et al. 1978]

A given query is decomposed into a sequence of subqueries $Q_{1}, Q_{2}, \ldots, Q_{p}$ with at most one variable in common between two consecutive subqueries, as in Wong and


AND (RI.B=R4.B)
AND(R4.C=R3.C)
AND(R4 $D=R 5 . D)\}$
(a)

(b)


(c)

(d)

Figure 18. Query processing in Baldissera et al. [1979].

Youssefi [1976]. Each subquery is irreducible. A query is irreducible if and only if its query-graph is a chain with two nodes or a cycle with $k$ nodes and all its equivalent query-graphs have a cycle with the same $k$ nodes, where $k \geq 3$. For example, given a query as shown in Figure 19a, the algorithm decomposes the query into two irreducible subqueries $Q_{1}$ followed by $Q_{2}$, as shown in Figure 19b. $Q_{1}$ is processed and the result is incorporated into the query graph of $Q_{2}$, which is then processed.
Distributed INGRES [Epstein et al. 1978] considers both data communication
and local processing costs and allows relations to be fragmented in various sites (see Section 9). For ease of presentation, it is assumed that relations are not fragmented, and only the data communication cost is considered. If a subquery is a chain with two nodes, say $R_{x}$ and $R_{y}$, then either $R_{x}$ is sent to the site containing $R_{y}$, or $R_{y}$ is sent to the site containing $R_{x}$, depending on which strategy incurs less cost. If the subquery is a cyclic query, then a decision has to be made whether to process the entire subquery at once or subdivide it into pieces. The subquery is subdivided if it

$Q 1=\{R 1 . D \mid(R 1 A=R 2 . A)$
AND (R2.B=R3.B)
AND (R1.C = R3.C) \}



$$
\begin{aligned}
& Q=\{R 4 . E \mid(R 1 . A=R 2 . A) \\
& \text { AND }(R 1 . D=R 4 . D) \\
& \text { AND (R2.B=R3.B) } \\
&\text { AND (R3.C }=R 1 . C)\}
\end{aligned}
$$

(a)
$Q 2=\{R 4 . E \mid(R 123 . D=R 4 . D)\}$

where R123 is the result of processing Q1
(b)
$|R 1| \leq|R 2| \leq|R 3|$

process at once
(c)

Figure 19. Query processing in Epstein et al. [1978].
results in a lower cost. As an example, given three relations $R_{1}, R_{2}$, and $R_{3}$ with $\left|R_{1}\right| \leq$ $\left|R_{2}\right| \leq\left|R_{3}\right|, w_{1}=w_{2}=w_{3}$, and each relation residing in a different site, $\left|R_{t}\right|$ is the number of tuples of $R_{z}$ and $w_{t}$ is the average tuple width of $R_{r}$. Figure 19 c illustrates the strategy for processing the subquery $Q_{1}$ at once with minimum cost, while Figure 19d shows all possible strategies for subdividing $Q_{1}$ into pieces. The strategy in Figure 19c means that $R_{1}$ and $R_{2}$ are sent to the site containing $R_{3}$, and the answer to $Q_{1}$ is produced there. The first strategy, in Figure 19d, for instance, is interpreted as follows: (i) $R_{1}$ is sent to the site containing $R_{2}$ to perform the join of $R_{1}$ with $R_{2}$; (ii) the resulting relation, let us say $R_{12}$, is sent to the site containing $R_{3}$ to
perform the join with $R_{3}$. The cost of the minimal strategy in Figure 19d is compared with that of the strategy in Figure 19c. If the former strategy has less cost, $Q_{1}$ is then subdivided into the pieces as given by the minimal strategy; otherwise, $Q_{1}$ is processed at once.

It is important to point out that both the query processing algorithms in distributed INGRES and $\mathrm{R}^{*}$ take into consideration the local processing cost as well as the data transmission cost. Distributed INGRES also provides a different algorithm to optimize the response time of a query in a broadcasting system.

Wong [1981] suggested decomposing a given query into a sequence of subqueries that contain at most one join and possibly
some projections and selections. His method emphasizes maximizing parallelism by making use of redundancy of data.

## 9. FRAGMENT PROCESSING

A relation can be viewed as a matrix where the rows stand for tuples and the columns stand for attributes. A horizontal fragment of a relation is a subset of the rows of the matrix. It is obtained by applying a select operation on the relation. Sometimes a horizontal fragment is accessed frequently in one site, while another horizontal fragment is referenced frequently in another site. Thus it may be beneficial to assign fragments to sites according to their reference locality. A vertical fragment of a relation is a subset of the columns of the relation and is constructed by using the projection operation on the relation. In this section, we restrict our discussion to horizontal fragments.

In Goodman et al. [1979], a query that refers to fragmented relations is first decomposed into subqueries. The SDD-1 algorithm described in Section 7 is then used to obtain an answer for each subquery. The union of the answers of all the subqueries is the answer to the query.

The following procedure is used to decompose the query into subqueries: For a given query, (1) find all the fragmented relations referenced by the query, say $F, G$, $\ldots$, and $H$; (2) for each combination of fragments $F_{i}, G_{j}, \ldots$, and $H_{k}$, where $F_{i}, G_{j}$, $\ldots$, and $H_{k}$ are the fragments of $F, G, \ldots$, and $H$, respectively, construct a subquery by replacing $F, G, \ldots$, and $H$ in the query by $F_{\imath}, G_{j}, \ldots$, and $H_{k}$, respectively. Thus the number of subqueries is equal to the product of the numbers of fragments of the referenced relations.

Example 9.1. Let $F=\left\{F_{1}, F_{2}\right\}$ and $G=$ $\left\{G_{1}, G_{2}\right\}$ be two fragmented relations.

Let $Q=\{F . B \mid F . A=G . A\}$ be a query.
Query $Q$ is decomposed by the above procedure into the subqueries:

$$
\begin{aligned}
& Q_{1}=\left\{F_{1} \cdot B \mid F_{1} \cdot A=G_{1} \cdot A\right\}, \\
& Q_{2}=\left\{F_{2} \cdot B \mid F_{2} \cdot A=G_{1} \cdot A\right\}, \\
& Q_{3}=\left\{F_{1} \cdot B \mid F_{1} \cdot A=G_{2} \cdot A\right\}, \\
& Q_{4}=\left\{F_{2} \cdot B \mid F_{2} \cdot A=G_{2} \cdot A\right\} .
\end{aligned}
$$

The four subqueries are then individually evaluated by SDD-1's query processing algorithm, and the union of the answers is the final answer.

A semijoin in a fragmented database environment will fall into one of the following three types: $F-F, R-F$, or $R-R$ [Chang 1982b]. An $F-F$ semijoin is one in which both the sending and the reduced relations are fragments; An $R-F$ 's sending relation is a whole relation, but the reduced relation is a fragment; in an $R-R$ semijoin, both relations are whole relations. Thus a queryprocessing algorithm in a fragmented database environment can be classified into three categories: $F-F$ semijoin-based algorithm, $R-F$ semijoin-based algorithm, and $R-R$ semijoin-based algorithm.

Version 1 of SDD-1's query-processing algorithm [Goodman et al. 1979] as described in Example 9.1 is an $F-F$ semijoinbased algorithm. An $R-F$ semijoin-based algorithm is introduced in Chang [1982b]. It repeatedly chooses a beneficial $R-F$ semijoin until no beneficial $R-F$ semijoin exists. Then the reduced fragments/relations are sent to the assembly site to produce the answer. The query-processing algorithm in Yu et al. [1983] is an $R-R$ semijoin-based algorithm. For each given semijoin $R_{i}-A$ $\rightarrow R_{j}$, where $R_{i}$ and $R_{j}$ may or may not be fragmented, it selects a set of sites where the semijoin can be performed with minimum cost.

Another way to process a query referencing fragmented relations follows [Epstein et al. 1978; Stonebraker et al. 1982]: One fragmented relation is chosen, and other fragmented relations referenced by the query are replicated at the sites of the chosen fragmented relation. As an example, let a query reference $R_{1}$ and $R_{2}$. Suppose that $R_{1}$ contains fragments $F_{11}$ at site 1 and $F_{12}$ at site 2, and $R_{2}$ contains fragments $F_{21}$ at site $1, F_{22}$ at site 3 , and $F_{23}$ at site 4 . The algorithm may then choose $R_{1}$ to remain fragmented and replicate $R_{2}$ at sites 1 and 2. The latter operation is performed by sending $F_{21}$ to site 2, and sending both $F_{22}$ and $F_{23}$ to sites 1 and 2. After $R_{2}$ arrives at the sites, $R_{2}$ is joined with $F_{11}$ at site 1 and with $F_{12}$ at site 2. The union of the tuples at the two sites is the final answer. In

Epstein et al. [1978], the relation to remain fragmented is chosen such that the amount of data processed is minimized. In reality, the cost of accessing data depends on the supporting access path. For example, with the use of an index, access could be speeded up significantly. In Yu et al. [1984b], such consideration is given to minimize the cost.

It is clear that this method of processing fragments may require substantial data transfer. We believe that in a realistic environment fragments are not placed arbitrarily, and there are placement dependencies between the locations of certain sets of fragmented relations on certain attributes. For example, Students (student-id, courseid, . . .) and Courses (course-id, instructor, ...) are two fragmented relations with a fragment of each relation situated at each campus of a university. Since a student usually takes courses only from his or her own campus, the join of a Student fragment in a campus and a Course fragment in a different campus on the attribute course-id is null. In other words, the join of the relations Course and Student can be performed at local sites without data transfer. Formally, if $F_{i j}$ should be a fragment of relation $R_{r}$ at site $j$, a placement dependency between $R_{1}$ and $R_{2}$ on attribute $A$ will exist if the join of $F_{1 k}$ and $F_{2 t}$ on attribute $A$ is null for $k \neq t$.

A query may reference a number of fragmented relations that share placement dependencies among them on a certain set of attributes. It is desirable to determine whether the query can be processed without data transfer [Yu et al. 1984a]. (A dual problem is to determine the placement of fragments such that queries can be processed without data transfer. A solution of the dual problem is given in Wong and Katz [1983].)

First, one seeks two relations (among the referenced relations of the query) that have a placement dependency between them on a certain attribute, that attribute being one of the joining attributes of the two relations. If two such relations cannot be found, the query cannot be processed without data transfer. Otherwise, $L P_{1}$ becomes the set containing these two relations, which can be joined together without data transfer,
although in practice they need not be the first pair of relations to be operated on. If another relation referenced by the query has a placement dependency with a relation in $L P_{1}$ on a set of attributes that is a subset of the set of joining attributes of the query, then it is added to $L P_{1}$. This process is repeated until either all relations are added to $L P_{1}$, in which case the query can be processed without data transfer or some relations remain and the query cannot be so processed.

If a query references both fragmented and unfragmented relations, then the following condition is sufficient for the query to be processed without data transfer. The fragmented relations (if there are two or more such relations referenced by the query) should satisfy the condition of the algorithm given in the last paragraph, while a copy of the unfragmented relations remains at each of the sites containing the fragmented relations.

## 10. THE TRANSFORMATION APPROACH

Perhaps a more systematic way to process queries is the transformation approach [Ceri and Pelagatti 1984; Ullman 1980] given as follows. In this approach, there exists a set of rules, where each rule transforms a query expression into an equivalent expression. The idea is to apply these rules repeatedly to obtain an expression that can be evaluated with a small cost.

Typically, the resulting relation after applying a unary operator, like project or select, tends to be smaller than the original relation, while the resulting relation after applying a binary operator, like join or union, can be significantly larger than the original operands. If the operands are in different sites, it will be profitable to reduce their sizes by applying the unary operators while preserving the equivalence of the expressions. For example, joining the relations $R_{1}(A, B, C)$ and $R_{2}(B, E, F)$ on attribute $B$ and then projecting the result on the attributes $(A, B, E)$ is equivalent to projecting $R_{1}$ on attribute $A$ and $B$ to eliminate $C$, projecting $R_{2}$ on the attributes $B$ and $E$ to eliminate $F$, and then taking the join of the two reduced relations. If $R_{1}$ and

(a)

(b)

Figure 20. Two equivalent expression trees.

(b)

Figure 21. Fragment processing by eliminating unnecessary fragments.
$R_{2}$ are in different sites, the latter expression can be evaluated with less data transfer and is therefore preferable. The rule that is applicable in this case is

$$
\mathrm{U}\left(R_{1} \mathrm{~B} R_{2}\right)=\mathrm{U}\left(R_{1}\right) \mathrm{B} \mathrm{U}\left(R_{2}\right),
$$

where $U$ is a unary operator and $B$ is a binary operator. A complete set of rules and the conditions under which the rules are applicable can be found in Ceri and Pelagatti [1984] and Ullman [1980].

In general, an expression can be represented by an expression tree in which each binary operation between two operands is represented by a subtree where the operands are two nodes whose parent is the operation, and each unary operation on an operand is represented by a subtree with the operation as the parent of the operand (illustrated in Fig. 20a). The application of the rule transforms the expression tree into an equivalent expression tree given in Figure 20b. The strategy is to move the unary operators toward the leaves of the tree as much as possible. The evaluation of the expression tree starts from the leaves toward the root, so that the unary operators can be evaluated as quickly as possible, reducing the original relations to smaller ones. This strategy of reducing the sizes of
intermediate relations applies to both centralized and distributed databases. In centralized databases, the intermediate relations move between the secondary and the main memories, while the movement in distributed databases is between the sites. In each case, reducing the sizes of the intermediate results seem logical.

When a relation is fragmented and placed into two or more sites, an expression involving the relation can be rewritten as an expression involving the fragments of the relation. Figure 21 illustrates how relation $R_{1}$ is replaced by its fragments $F_{11}$ and $F_{12}$, where each fragment is defined by a condition on the attribute $A_{1}$. In Figure 21, a selection is applied to $R_{1}$ on the attribute $A_{1}$. Since only one of the fragments $F_{11}$ and $F_{12}$ can satisfy the selection condition $A_{1}=$ " $X$," the expression reduces to a selection of a single fragment. Thus, although the fragments are located at different sites, it is sufficient to perform a selection at the site containing the appropriate fragment; the transfer of the other fragment is not required.

It is easy to see that the earlier approaches are special cases of the transformation approach. For example, a semijoin is used to transform a given expression
involving a join into an equivalent expression including the semijoin operation. A rule in support of this process is

$$
R_{1} \text { join } R_{2}=R_{1} \text { join }\left(R_{1} \rightarrow R_{2}\right) .
$$

Thus $R_{2}$ can be reduced by the semijoin operation before the join with $R_{1}$ takes place.

Similarly, the fragment and replicate approach [Epstein et al. 1978] is supported by the rule

$$
R_{1} \text { join } \underset{\imath}{U} F_{2 t}=\underset{\imath}{U}\left(R_{1} \text { join } F_{2 t}\right)
$$

in which $R_{1}$ is joined with each individual fragment of $R_{2}$ and then the union is taken rather than assembling all fragments of $R_{2}$ into a site and then joining with $R_{1}$.
Not only can equivalence of expressions be captured by the rules, but semantic information can also be represented. For example, if a relation should give the facilities of ports and another relation the properties of ships, then the type of ships that can go to their respective type of ports can be expressed as a rule. Thus artificial intelligence techniques may also be applicable in the processing of queries (see, e.g., King [1982]). When the number of rules is large, it is difficult to choose the appropriate sequence of rules to be applied, and it can be a time-consuming process.

## 11. CONCLUSION

We assume in this paper that a relational database is used, and that queries are expressed in a QUEL-like tuple relational calculus. We did not cover those queryprocessing algorithms for aggregate queries [Kim 1982; Yu et al. 1984a] and quantified queries [Jarke and Koch 1983; Jarke and Schmit 1982]. We have sketched some of the ideas used in the existing distributed query-processing algorithms: the estimation of the size of intermediate relations, the use of semijoins, the separation of an algorithm based on semijoins into three phases, the properties of tree queries that allow them to be processed rather efficiently, the transformation of cyclic queries into tree queries, the enhancement of semijoin strategies, the enumeration of strategies, and the different ways of han-
dling fragments. The transformation approach can be viewed as a generalization of some of the ideas presented here. We hope that large-scale experiments will be conducted to verify the usefulness of these ideas.

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